Circuits Integres Mixtes Analogique-Numerique

Conversion Analogique-Numerique ΣΔ

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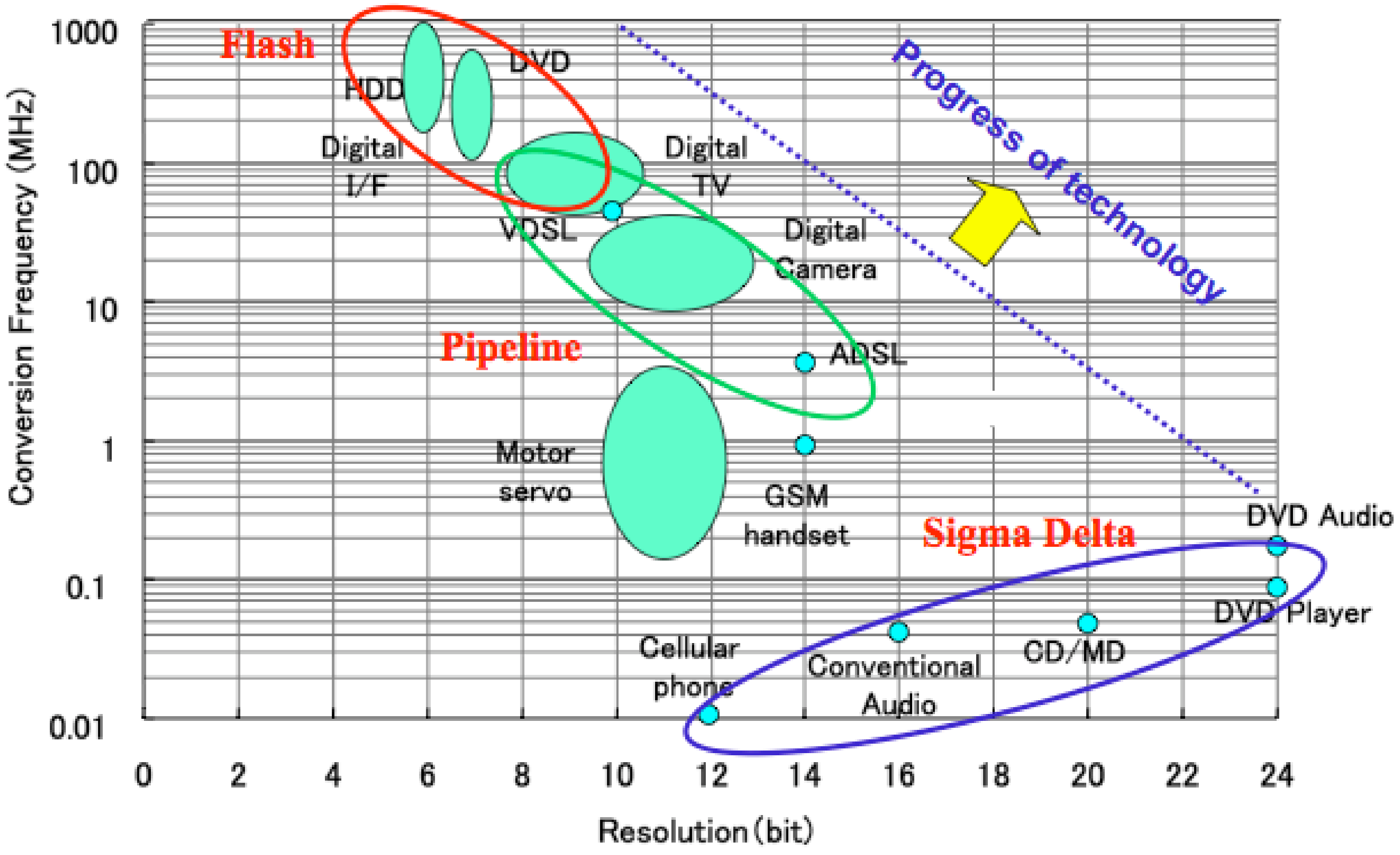
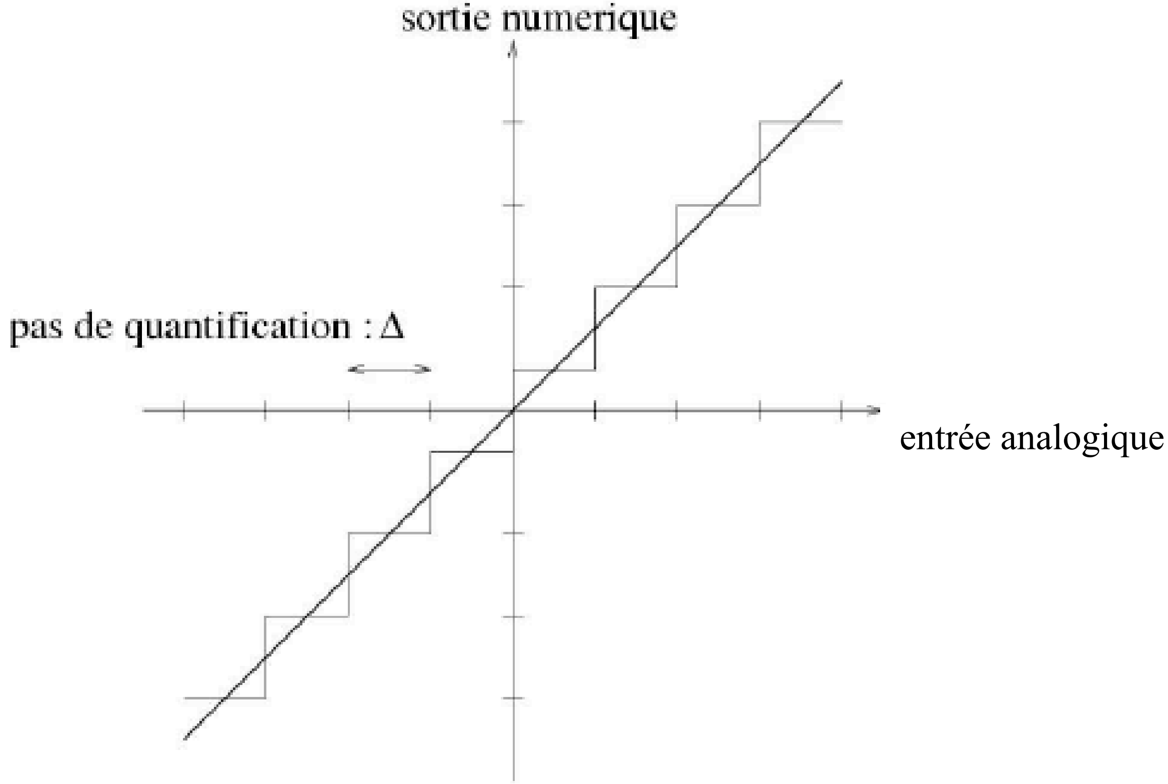
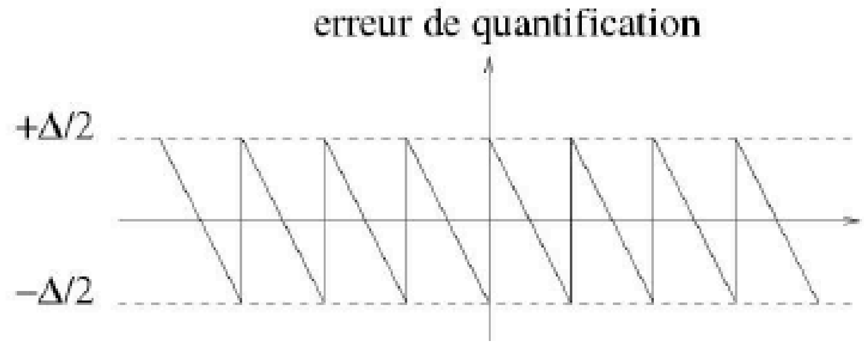
Performances and applications of ADCs

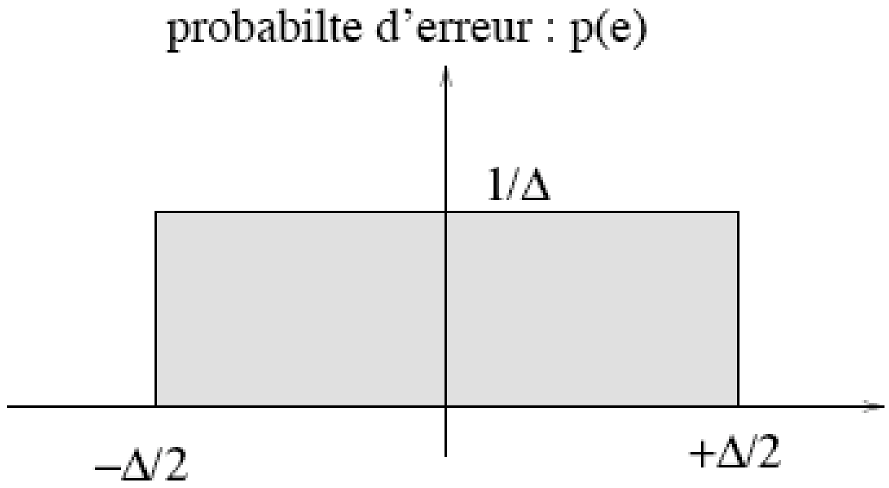


La Quantification





Bruit de Quantification



La variance du bruit de quantification :

$$\sigma_q^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2 dq = \frac{\Delta^2}{12}$$

La densité spectrale de puissance du bruit de quantification :

$$|E(f)|^2 = \frac{\sigma_q^2}{f_s} = \frac{\Delta^2}{12 f_s}$$

La puissance du bruit de quantification dans la bande utile :

$$P_q = \int_{-f_m}^{f_m} |E(f)|^2 df = \frac{\Delta^2}{12 OSR} \qquad \text{où} \quad OSR = \frac{f_s}{2 f_m}$$

Rapport Signal sur Bruit

$$\Delta = \frac{2.A}{2^N} = \frac{A}{2^{N-1}}$$

$$SNR = \frac{P_{signal}}{P_q} = \frac{\frac{A^2}{2}}{\frac{\Delta^2}{12OSR}}$$

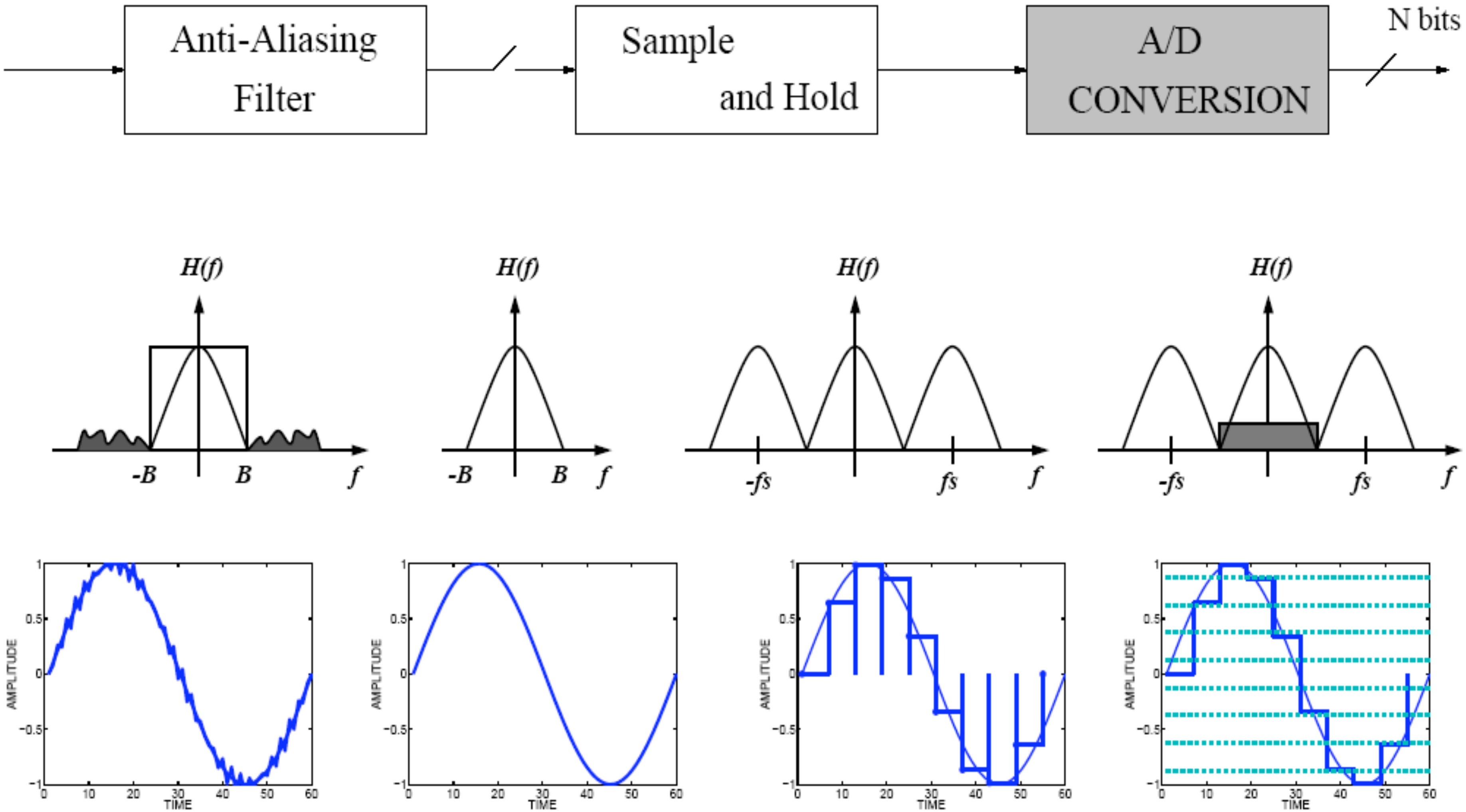
$$SNR\,[dB] = 10\log 2^{2N-3} * 12OSR$$

$$SNR\,[dB] = 1.76 + 6.02N + 10\log OSR$$

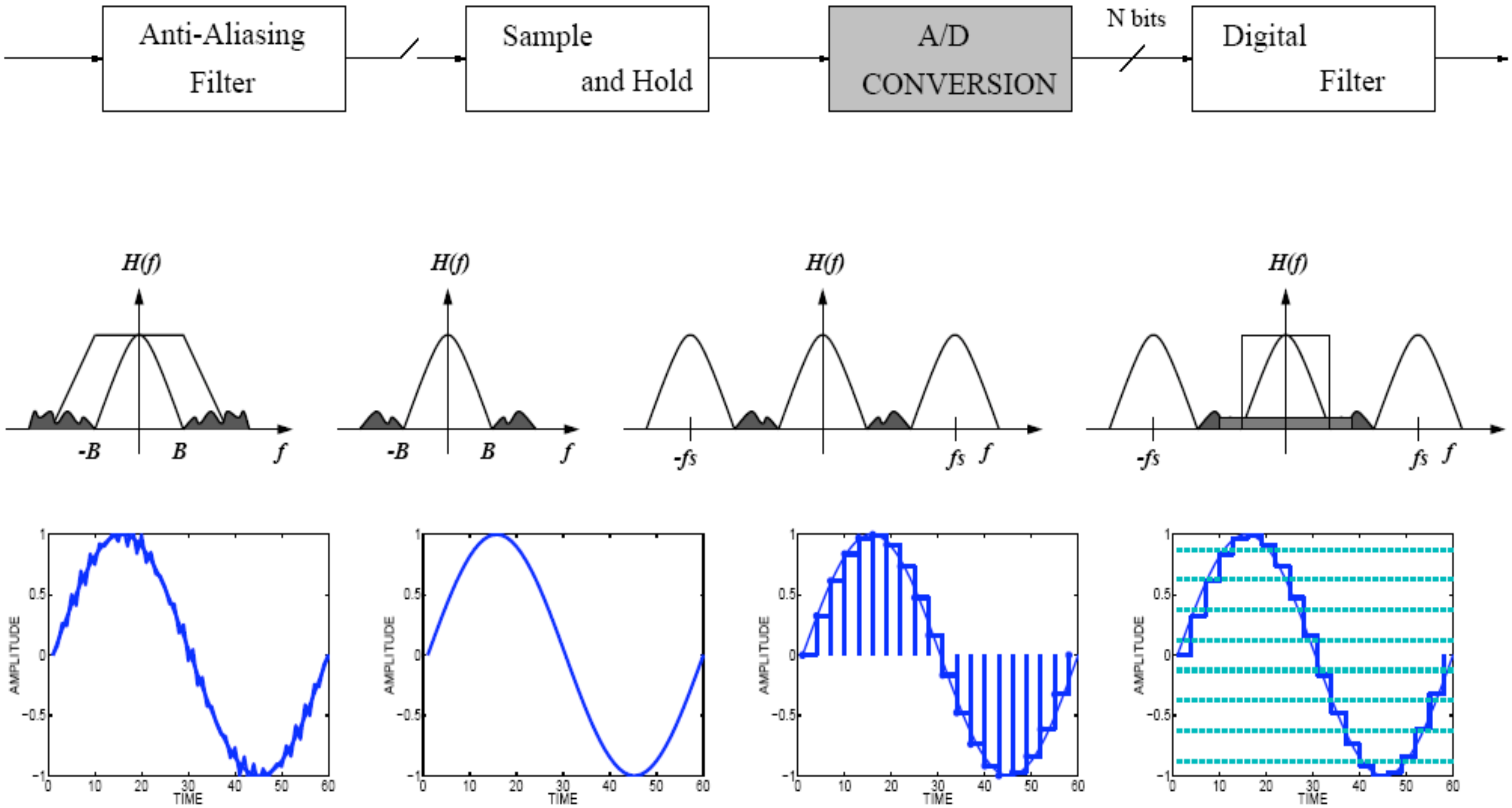
$$OSR \times 2 \Rightarrow SNR_{dB} + 3dB$$

$$\Rightarrow +0.5 \text{ bit of resolution}$$

A/D Conversion

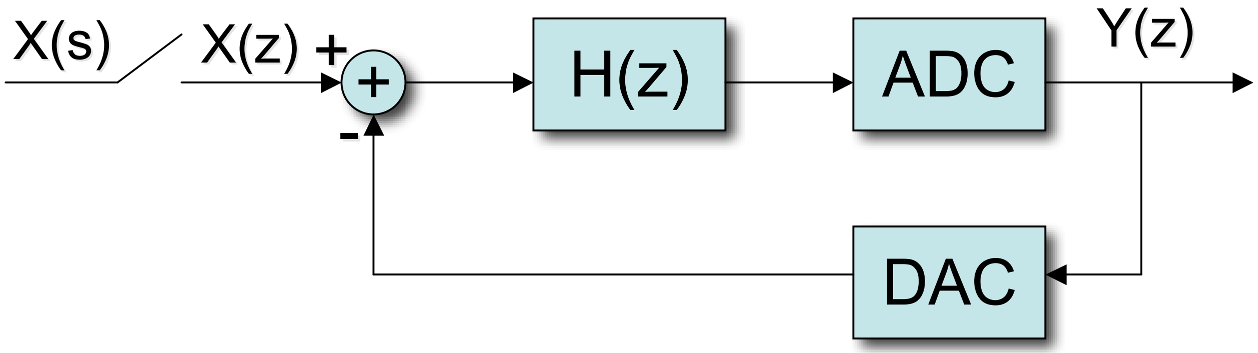


Oversampled A/D Conversion



Noise Shaping

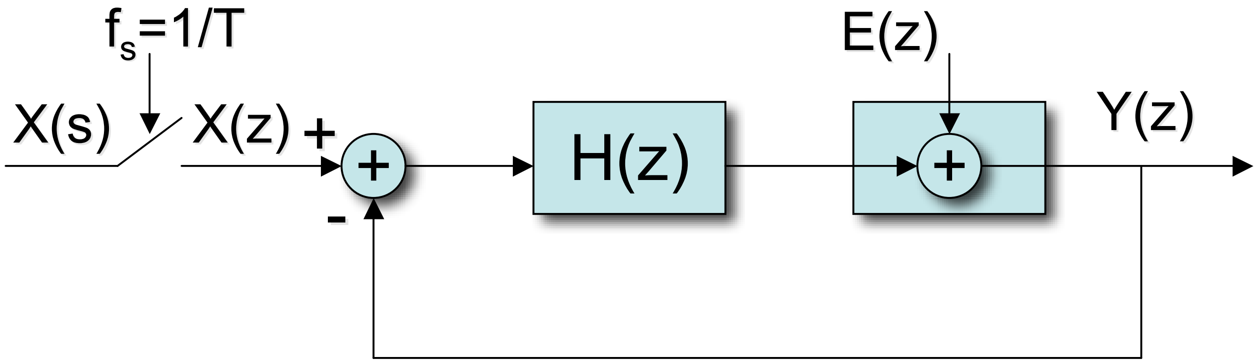
The ADC is put in a feedback loop with a filter H(z):



• The continuous-time analog signal, X(s), is sampled at the input to obtain a discrete-time signal, X(z).
• H(z) is a discrete-time analog filter, usually implemented in the switched capacitors technique.
• Y(z) is the discrete-time digital output signal.

Under certain conditions:
• The ADC can be modeled as a source of quantization noise.
• Ideally, in a discrete-time system the DAC does not add error.

Linear Mathematical Model



$$Y(z) = E(z) + H(z)\left[X(z) - Y(z)\right]$$

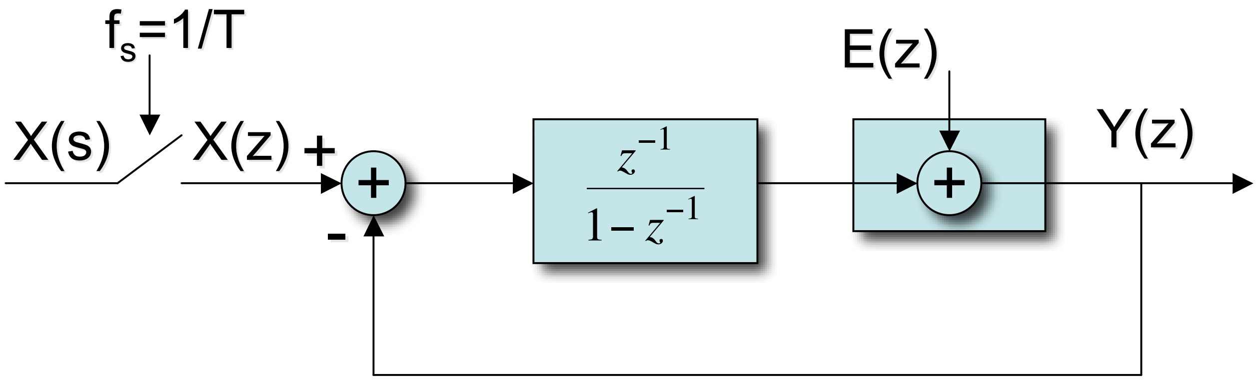
$$\left[1 + H(z)\right]Y(z) = E(z) + H(z)X(z)$$

$$\boxed{Y(z) = \frac{1}{1+H(z)}E(z) + \frac{H(z)}{1+H(z)}X(z)}$$

Noise Transfer Function: $$NTF(z) = \frac{Y(z)}{E(z)} = \frac{1}{1+H(z)}$$

Signal Transfer Function: $$STF(z) = \frac{Y(z)}{X(z)} = \frac{H(z)}{1+H(z)}$$

1st order ΣΔ Modulator



$$Y(z) = \frac{1}{1+H(z)}E(z) + \frac{H(z)}{1+H(z)}X(z)$$

$$H(z) = \frac{z^{-1}}{1-z^{-1}}$$

$$Y(z) = (1-z^{-1})E(z) + z^{-1}X(z)$$

$$Y(z) = E(z) - z^{-1}E(z) + z^{-1}X(z)$$

Inverse z Transform

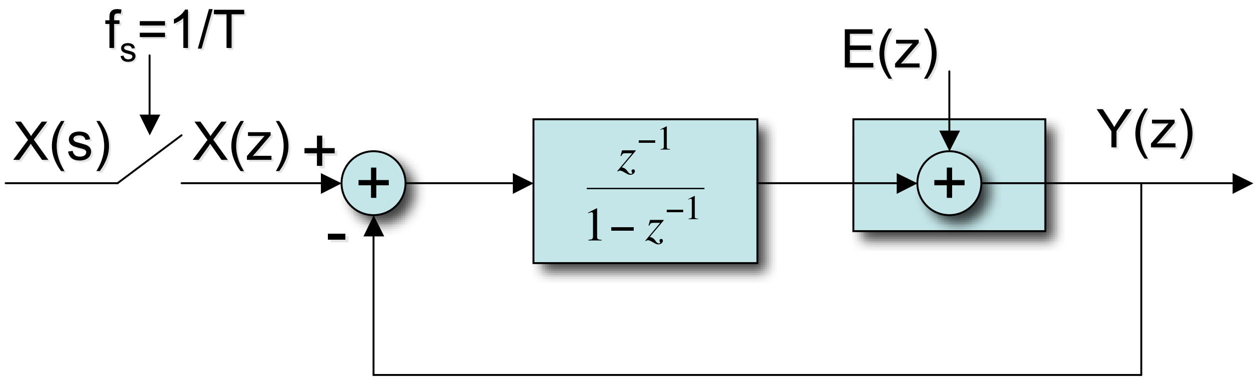
$$y[n] = e[n] - e[n-1] + x[n-1]$$

Quantization error $\quad q[n] = e[n] - e[n-1]$

$$if \quad f_{in} = 0 \quad \Rightarrow \quad e[n] = e[n-1] \quad \Rightarrow \quad q[n] = 0$$

$$as \quad f_{in} \uparrow \quad \Rightarrow \quad e[n] \neq e[n-1] \quad \Rightarrow \quad q[n] \uparrow$$

1st order ΣΔ Modulator



$$Y(z) = \frac{1}{1+H(z)}E(z) + \frac{H(z)}{1+H(z)}X(z)$$

$$H(z) = \frac{z^{-1}}{1-z^{-1}}$$

$$Y(z) = (1-z^{-1})E(z) + z^{-1}X(z)$$

Noise Transfer Function: $$NTF(z) = \frac{Y(z)}{E(z)} = 1 - z^{-1}$$

Signal Transfer Function: $$STF(z) = \frac{Y(z)}{X(z)} = z^{-1}$$

1st order ΣΔ Noise Transfer Function

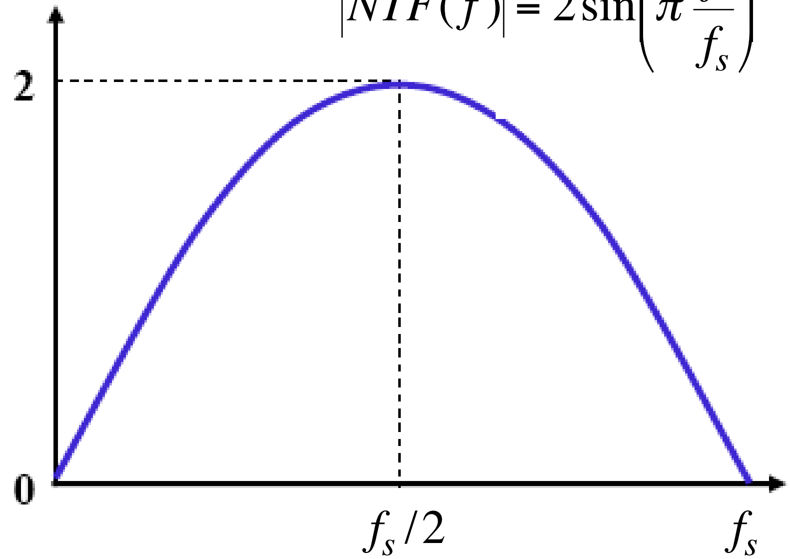
let $z \rightarrow e^{j\omega T}$

$$NTF(z) = 1 - z^{-1}$$

$$NTF(\omega) = 1 - e^{-j\omega T} = 2j \quad e^{-j\omega\frac{T}{2}} \quad \frac{e^{j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}}}{2j}$$

$$NTF(\omega) = 2j \quad e^{-j\omega\frac{T}{2}} \quad \sin\left(\omega\frac{T}{2}\right)$$

$$\left|NTF(f)\right| = 2\sin\left(\pi\frac{f}{f_s}\right)$$

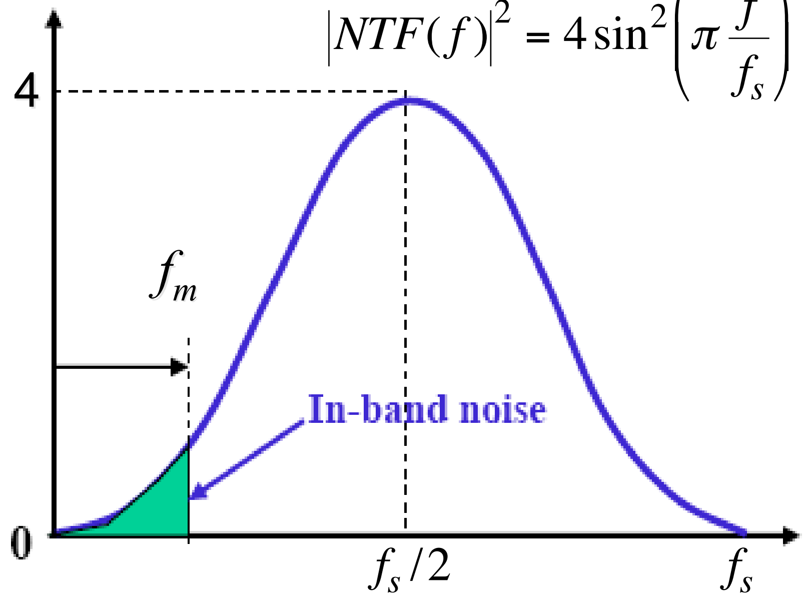


$$\begin{aligned}\text{In-Band Noise Power} &= \int_{-f_m}^{f_m} \left|E(f)\right|^2 \times \left|NTF(f)\right|^2 df \\ &= \int_{-f_m}^{f_m} \frac{\Delta^2}{12 f_s} \times 4\sin^2\left(\pi\frac{f}{f_s}\right) df \\ &= \frac{\Delta^2}{12}\frac{\pi^2}{3}\left(\frac{2f_m}{f_s}\right)^3 \quad \boxed{= \frac{\Delta^2}{12}\frac{\pi^2}{3}\frac{1}{OSR^3}}\end{aligned}$$

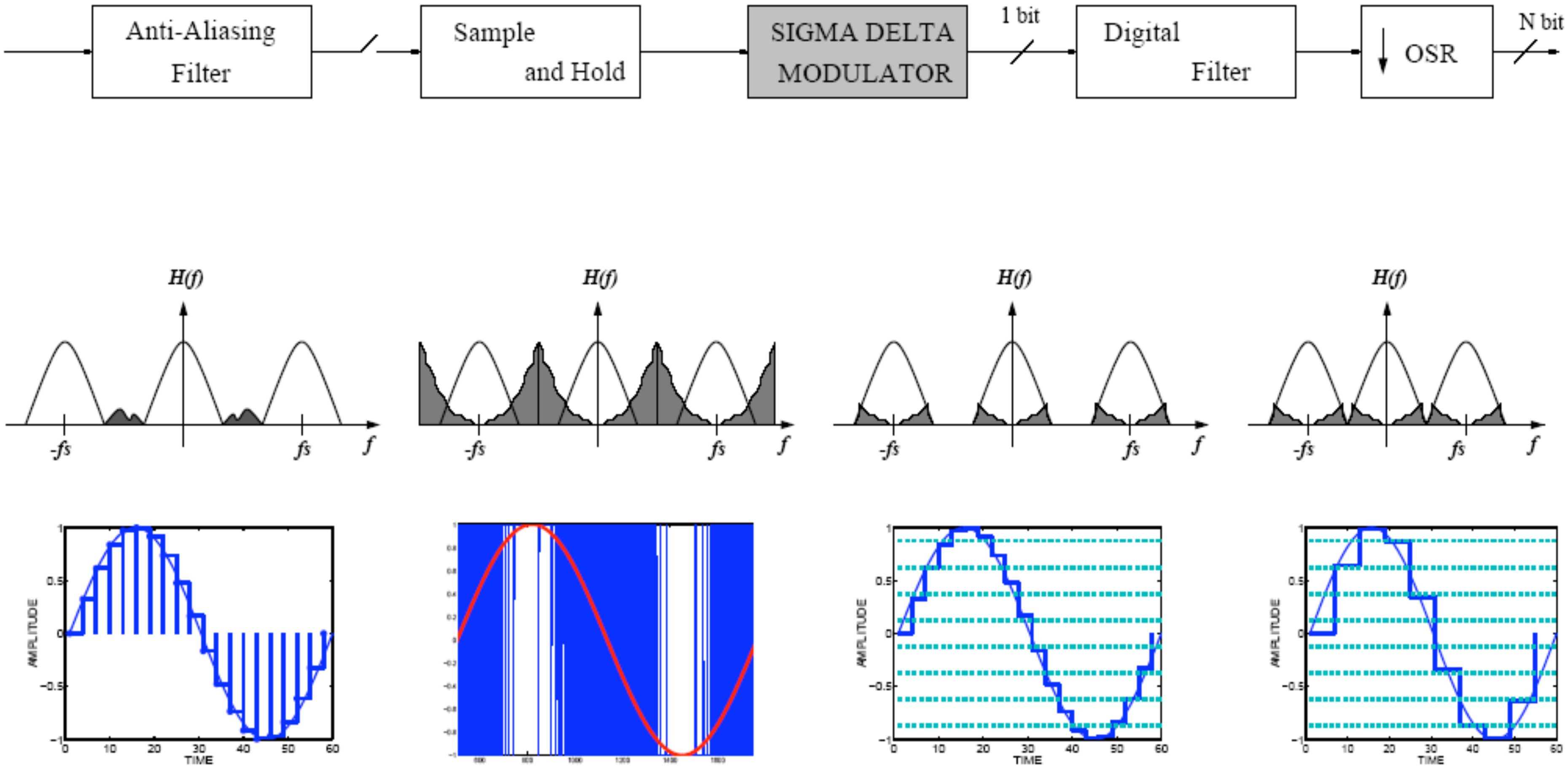
$$OSR \times 2 \Rightarrow SNR_{dB} + 9dB$$

$$\Rightarrow +1.5 \text{ bit of resolution}$$

$$\left|NTF(f)\right|^2 = 4\sin^2\left(\pi\frac{f}{f_s}\right)$$



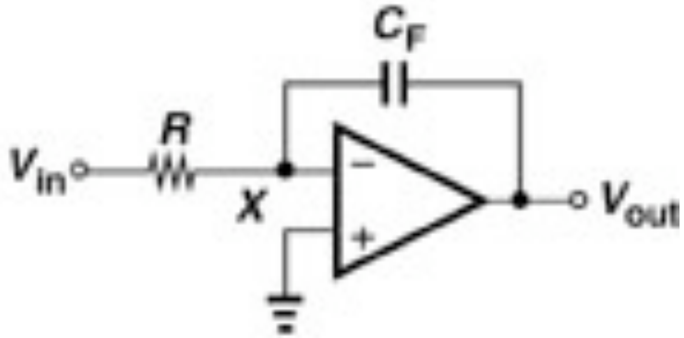
Noise-Shaped Oversampled A/D Conversion



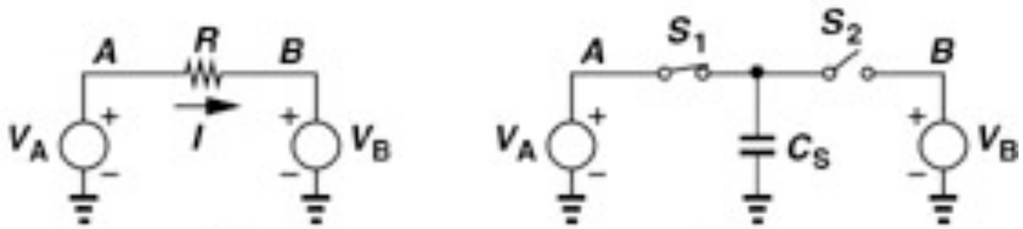
Integrator Circuit

Continuous-Time Integrator

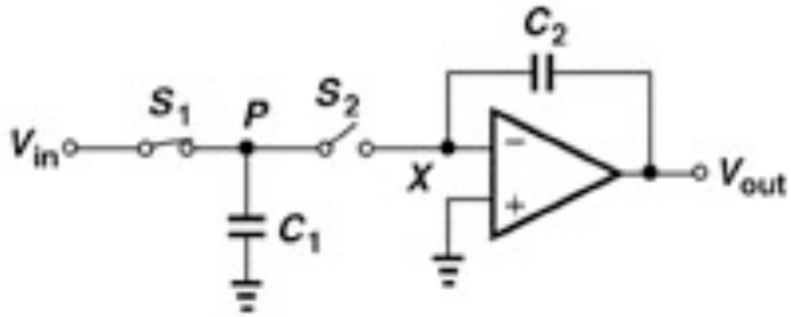
$$H(s) = \frac{Vout(s)}{Vin(s)} = \frac{-1}{s\,RC}$$



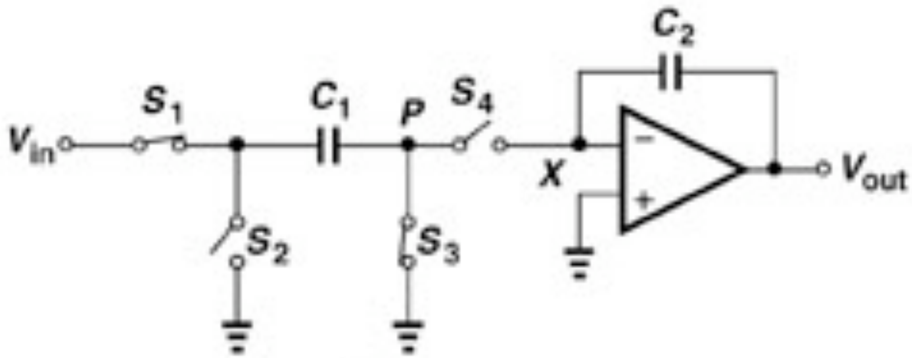
Continuous-Time and Discrete-Time Resistor



(a) (b)

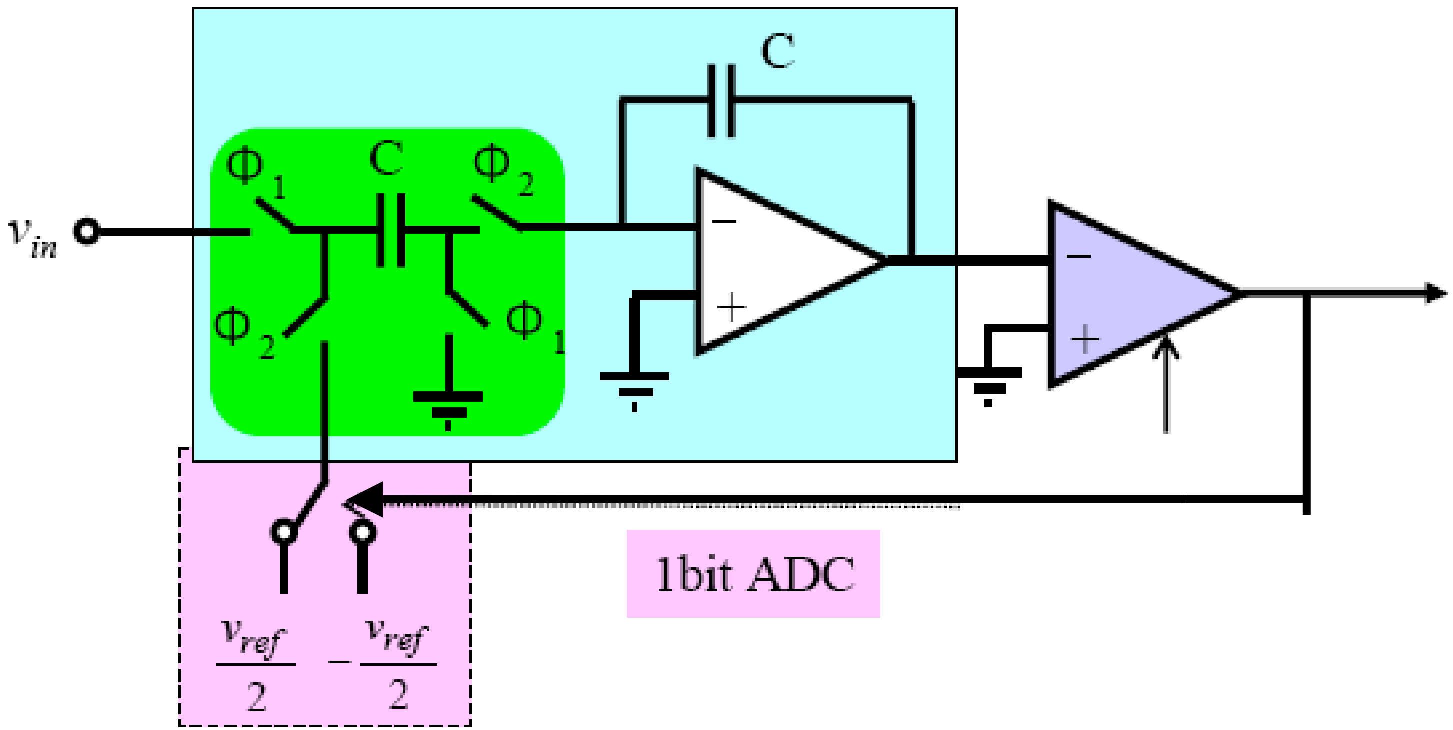


(a)

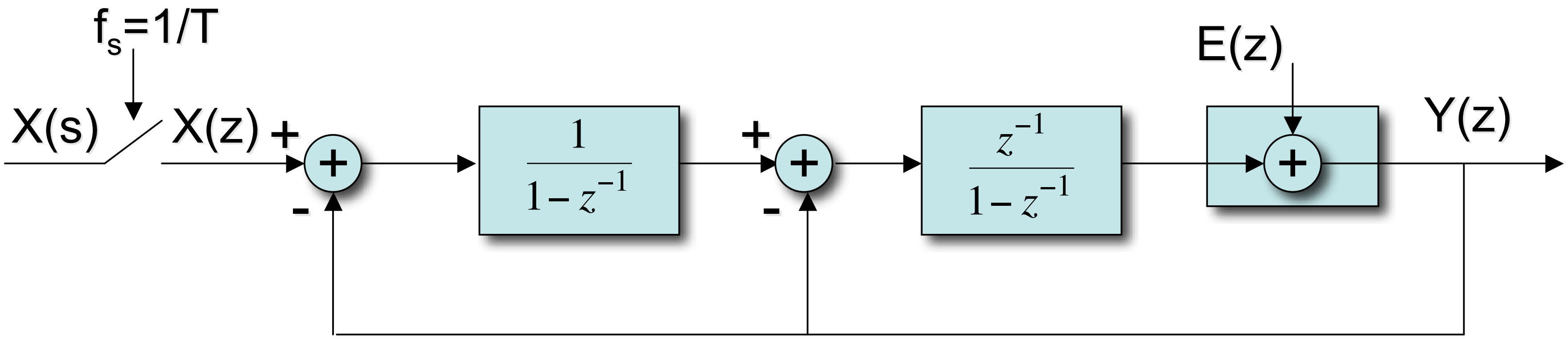


(a)

1st order ΣΔ using Switched Capacitors



2nd order ΣΔ Modulator



$$Y(z) = (1 - z^{-1})^2 E(z) + z^{-1} X(z)$$

Noise Transfer Function: $$NTF(z) = \frac{Y(z)}{E(z)} = \left(1 - z^{-1}\right)^2$$

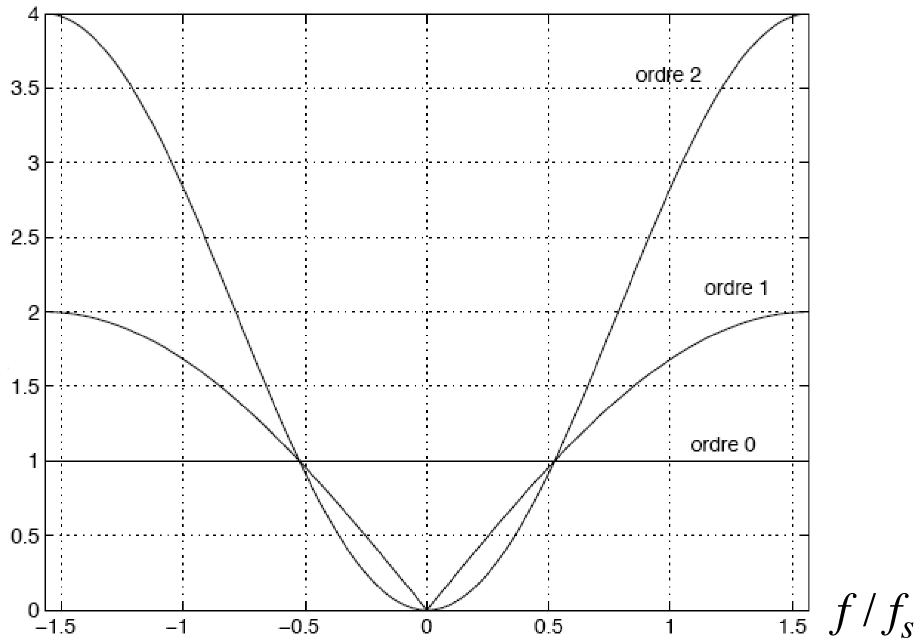
Signal Transfer Function: $$STF(z) = \frac{Y(z)}{X(z)} = z^{-1}$$

2nd order ΣΔ Noise Transfer Function

$$|NTF(f)| = 4\sin^2\left(\pi \frac{f}{f_s}\right)$$

$$\text{In-Band Noise Power} = \int_{-f_m}^{f_m} |E(f)|^2 \times |NTF(f)|^2 df$$

$$= \frac{\Delta^2}{12}\frac{\pi^4}{5}\frac{1}{OSR^5}$$



$OSR \times 2 \Rightarrow SNR_{dB} + 15dB$

$\Rightarrow +2.5$ bits of resolution

SNR of Ideal ΣΔ Modulators

In-Band Noise Power:

Conventional ADC:

$$\frac{\Delta^2}{12}\frac{1}{OSR}$$

1st Order ΣΔ:

$$\frac{\Delta^2}{12}\frac{\pi^2}{3}\frac{1}{OSR^3}$$

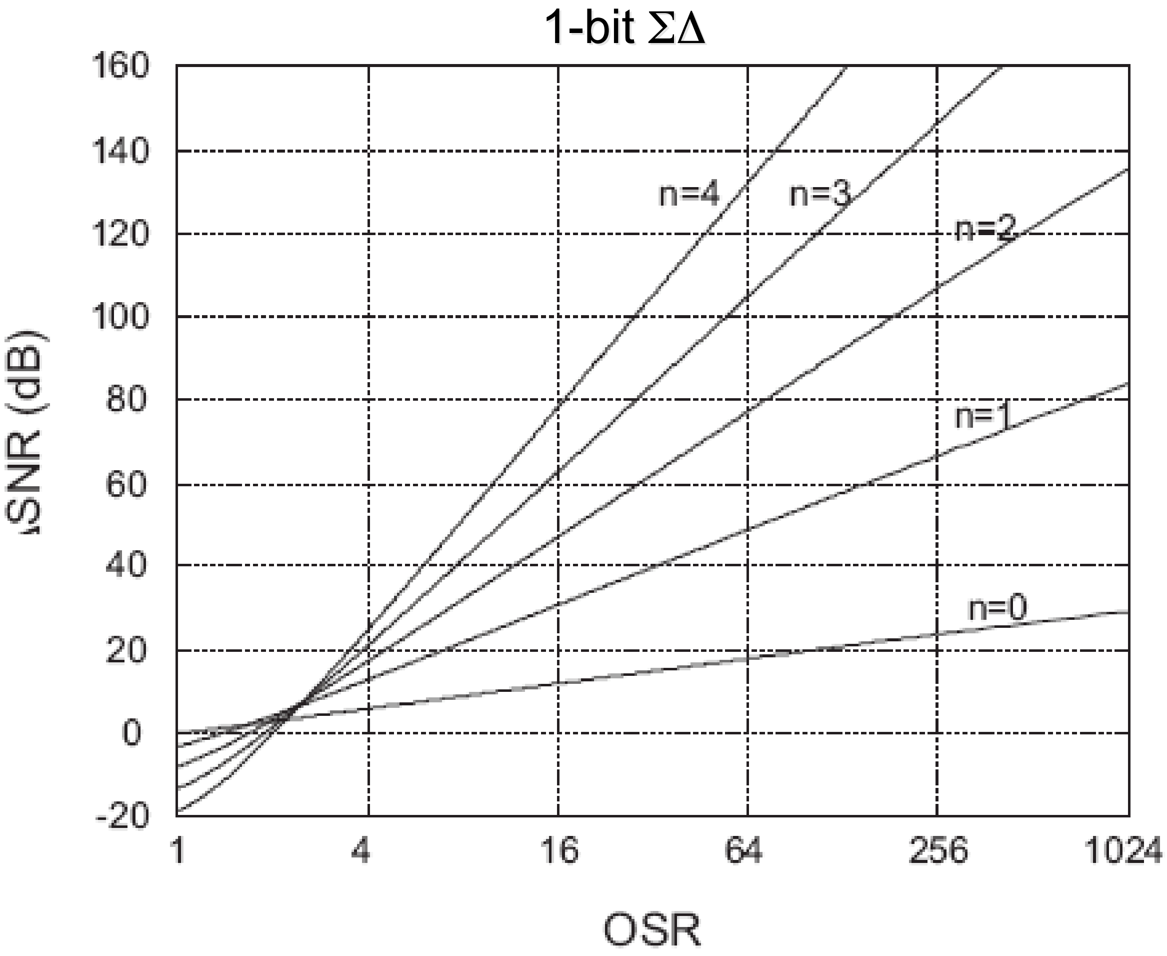
2nd Order ΣΔ:

$$\frac{\Delta^2}{12}\frac{\pi^4}{5}\frac{1}{OSR^5}$$

nth Order ΣΔ ??

$$\frac{\Delta^2}{12}\frac{\pi^{2n}}{2n+1}\frac{1}{OSR^{2n+1}}$$

Not Really !!



SNR of Ideal ΣΔ Modulators

In-Band Noise Power:

Conventional ADC:

$$\frac{\Delta^2}{12}\frac{1}{OSR}$$

1st Order ΣΔ:

$$\frac{\Delta^2}{12}\frac{\pi^2}{3}\frac{1}{OSR^3}$$

2nd Order ΣΔ:

$$\frac{\Delta^2}{12}\frac{\pi^4}{5}\frac{1}{OSR^5}$$

nth Order ΣΔ ??

$$\frac{\Delta^2}{12}\frac{\pi^{2n}}{2n+1}\frac{1}{OSR^{2n+1}}$$

Not Really !!

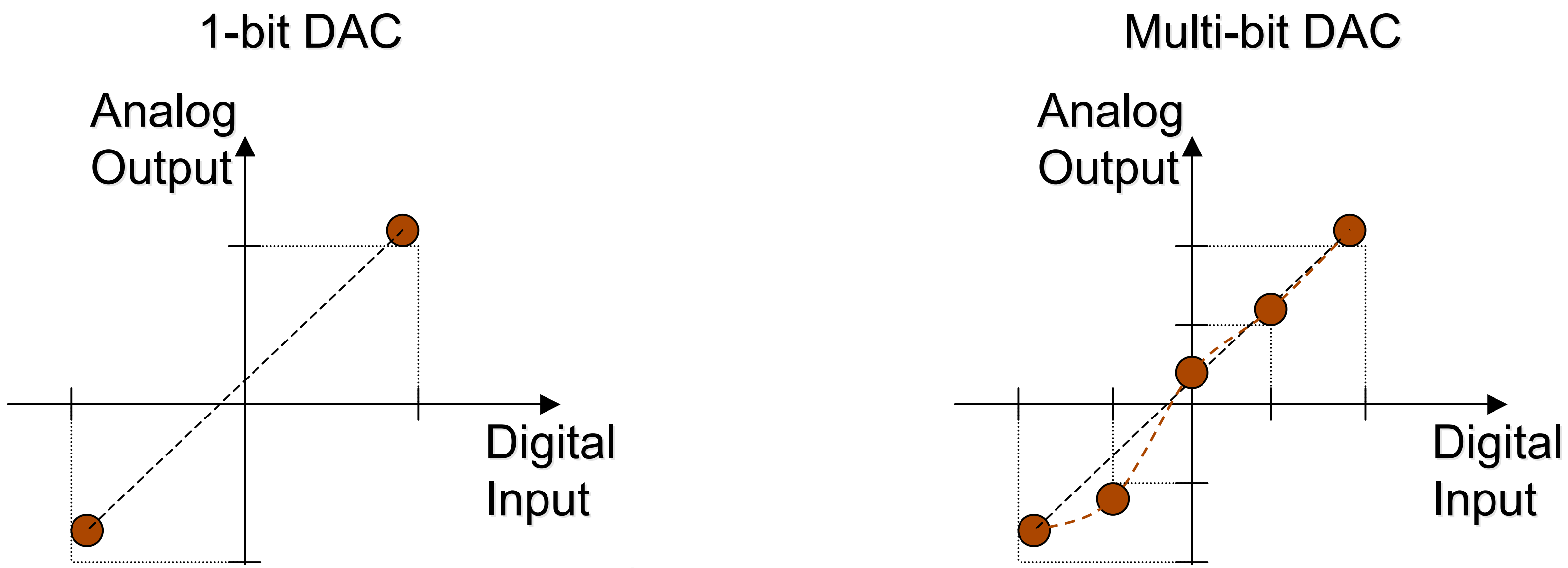
For a High SNR:

- Increase oversampling ratio, OSR ↑
- Increase order of noise-shaping function, n ↑
- Increase number of bits in the quantizer, Δ ↓

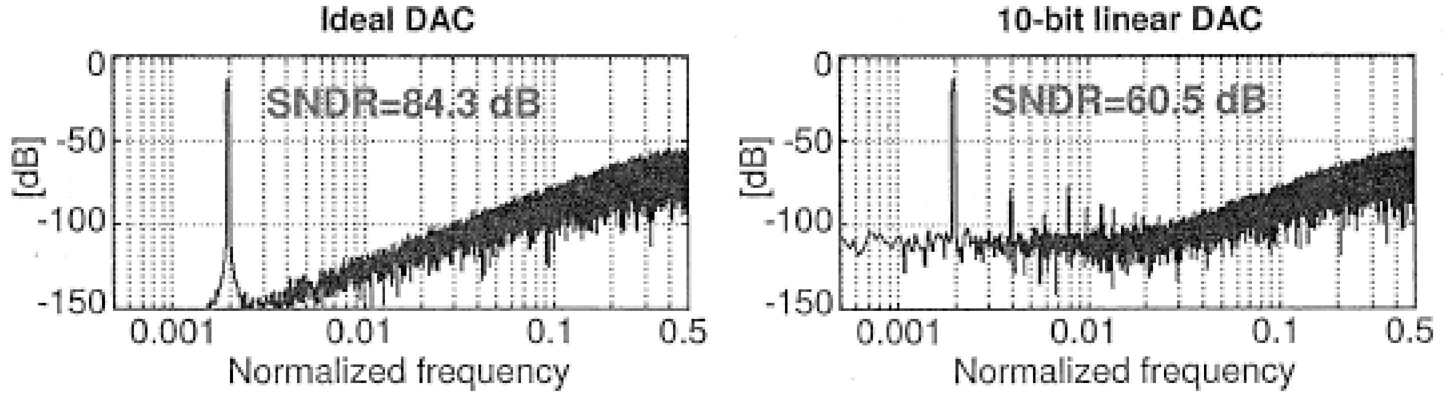
Non-Idealities:

- the sampling frequency is limited
- n>2 => stability problems
- # bits in the quantizer > 1 => Feedback DAC linearity problems

DAC Linearity



- Second-order ΔΣ modulator with 4-bit quantizer and DAC
- OSR = 32 ⇒ Ideal SNDR ≈ 14 bit



- Nonlinear DAC causes higher noise floor and harmonics.
- 10-bit linear DAC causes 10-bit level SNDR.

Improving DAC Linearity

1. Element calibration:

- During fabrication (e.g., laser trimming) – expensive, not effective for long-term process variations (temperature, aging, etc).
- During circuit operation – can be performed periodically, but increases analog design difficulty .

2. Dynamic element matching (DEM):

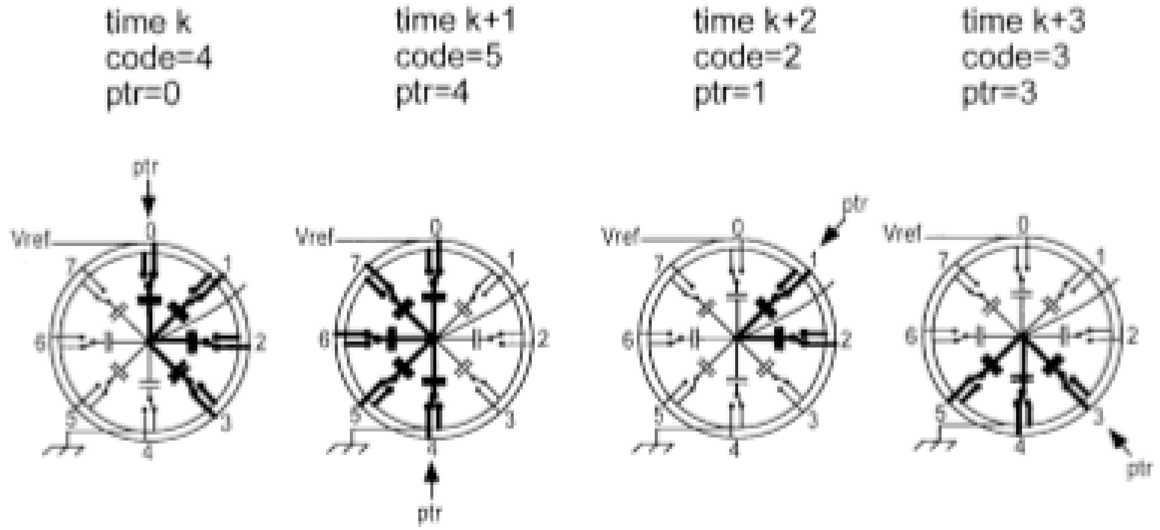
- Randomize usage of analog elements, so that DAC errors are averaged.
- Many different flavors are available (barrel-shifting, individual-level averaging, data-weighted averaging, tree-structure, etc).
- Works well, but only for high OSR (OSR > 16).

3. Digital Estimation and Correction of DAC errors:

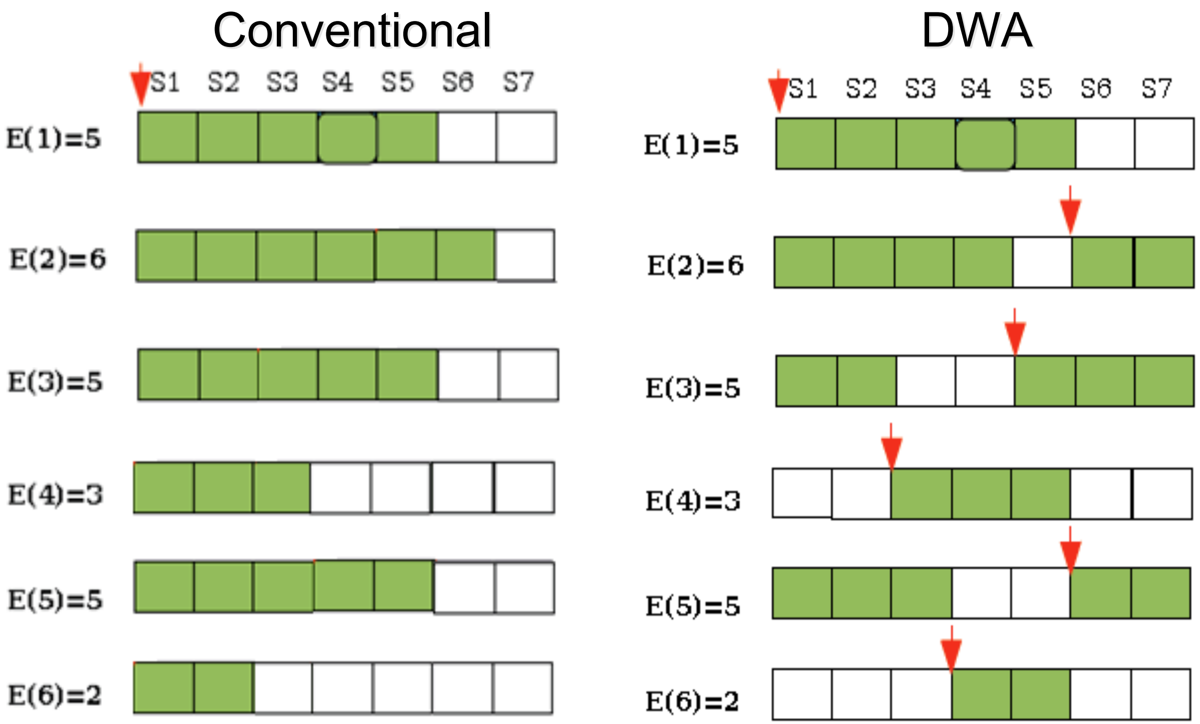
- Correlation based method. Works for any OSR.

Data Weighted Averaging

Switched-Capacitors DAC:



Current Sources DAC:



System model for stability analysis

Considering gains of integrator and quantizer.

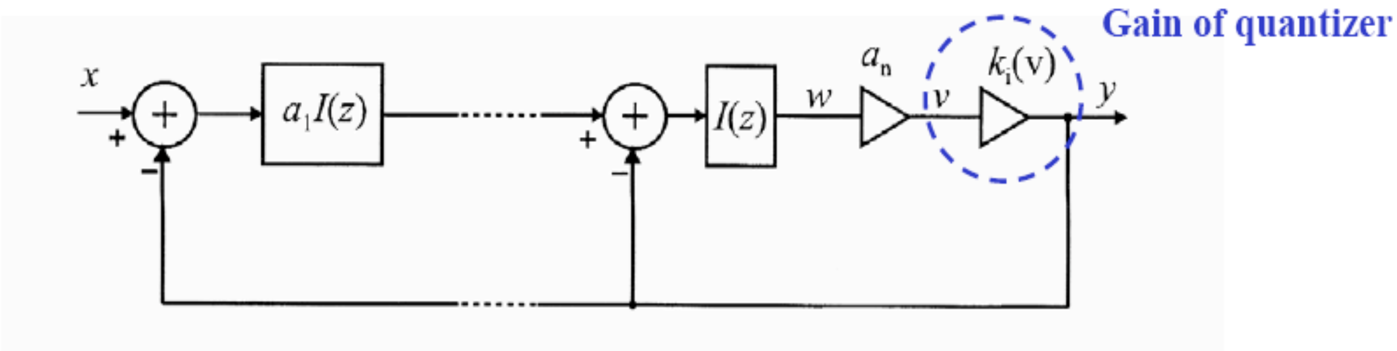
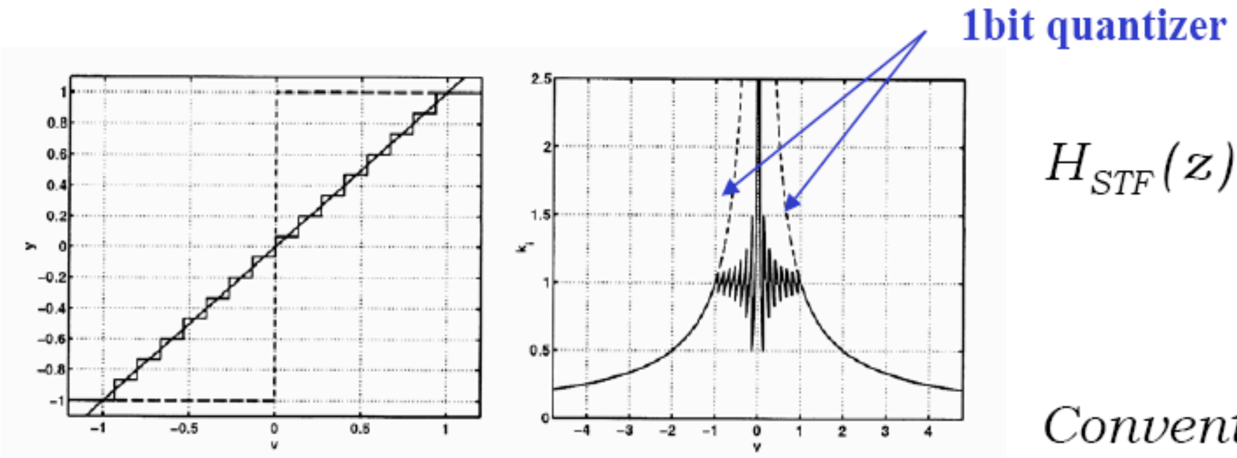


Figure 2.14: *Model of the ΔΣ loop with a variable quantizer gain* k_i.



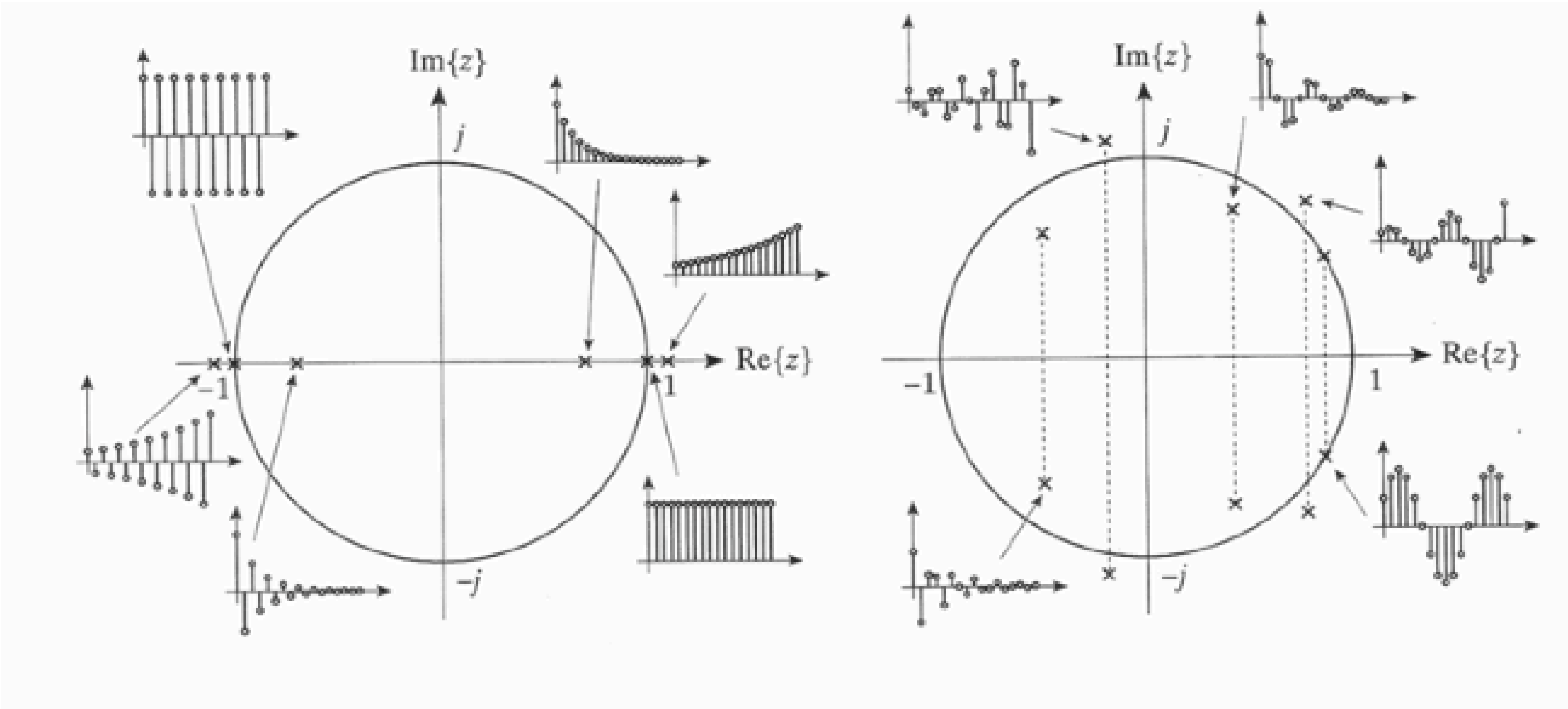
(a) Transfer function (b) Instantaneous quantizer gain k_i

$$H_{STF}(z) = \frac{k_i \cdot I^n(z) \cdot \prod_{i=1}^{n} a_i}{1 + k_i \cdot \sum_{i=1}^{n} \prod_{j=i}^{n} a_i I^{n-i+1}(z)}$$

$$Conventionally \quad I(z) = \frac{z^{-1}}{1 - z^{-1}}$$

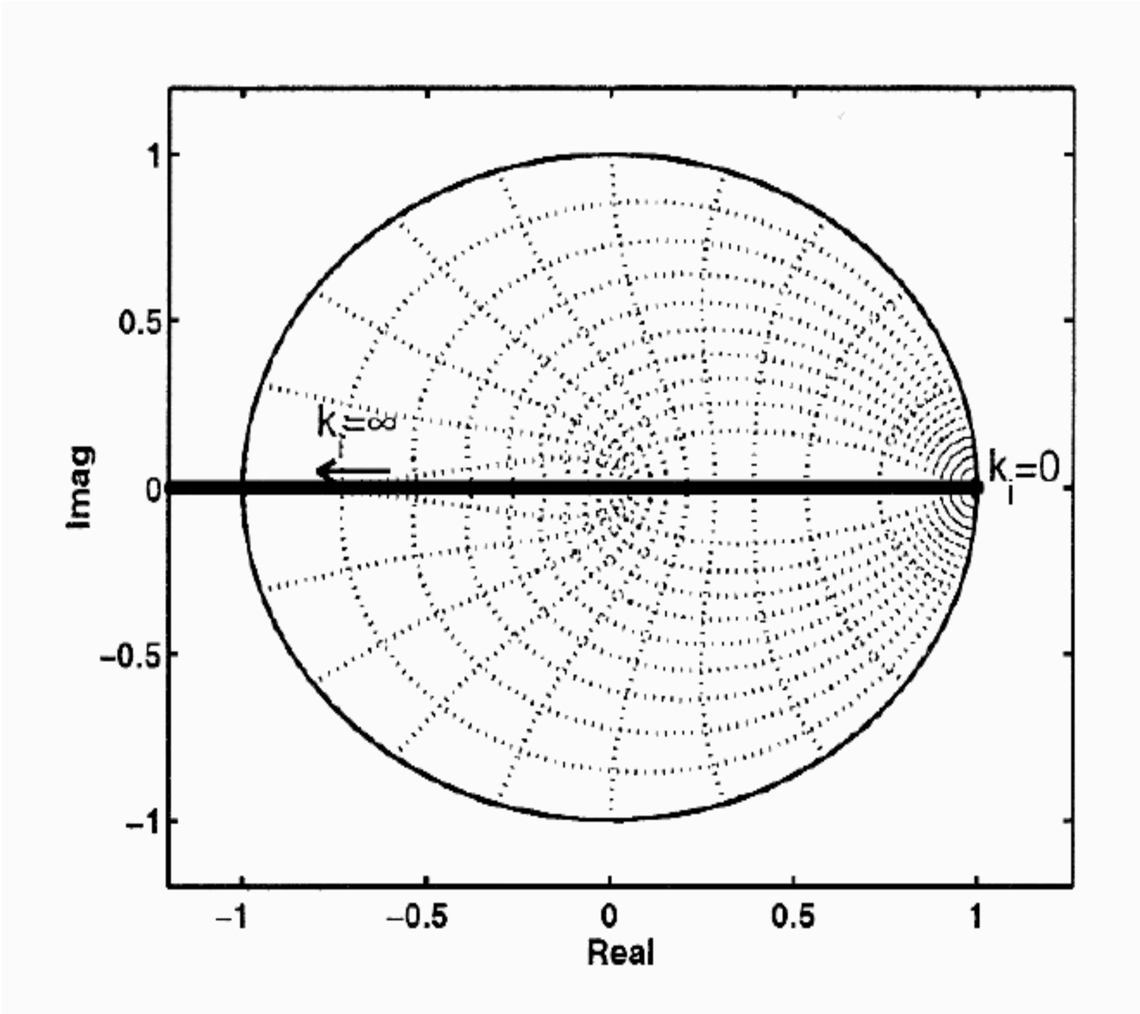
Stability and pole position

System is stable, if the poles locate on inside of the unit circle.



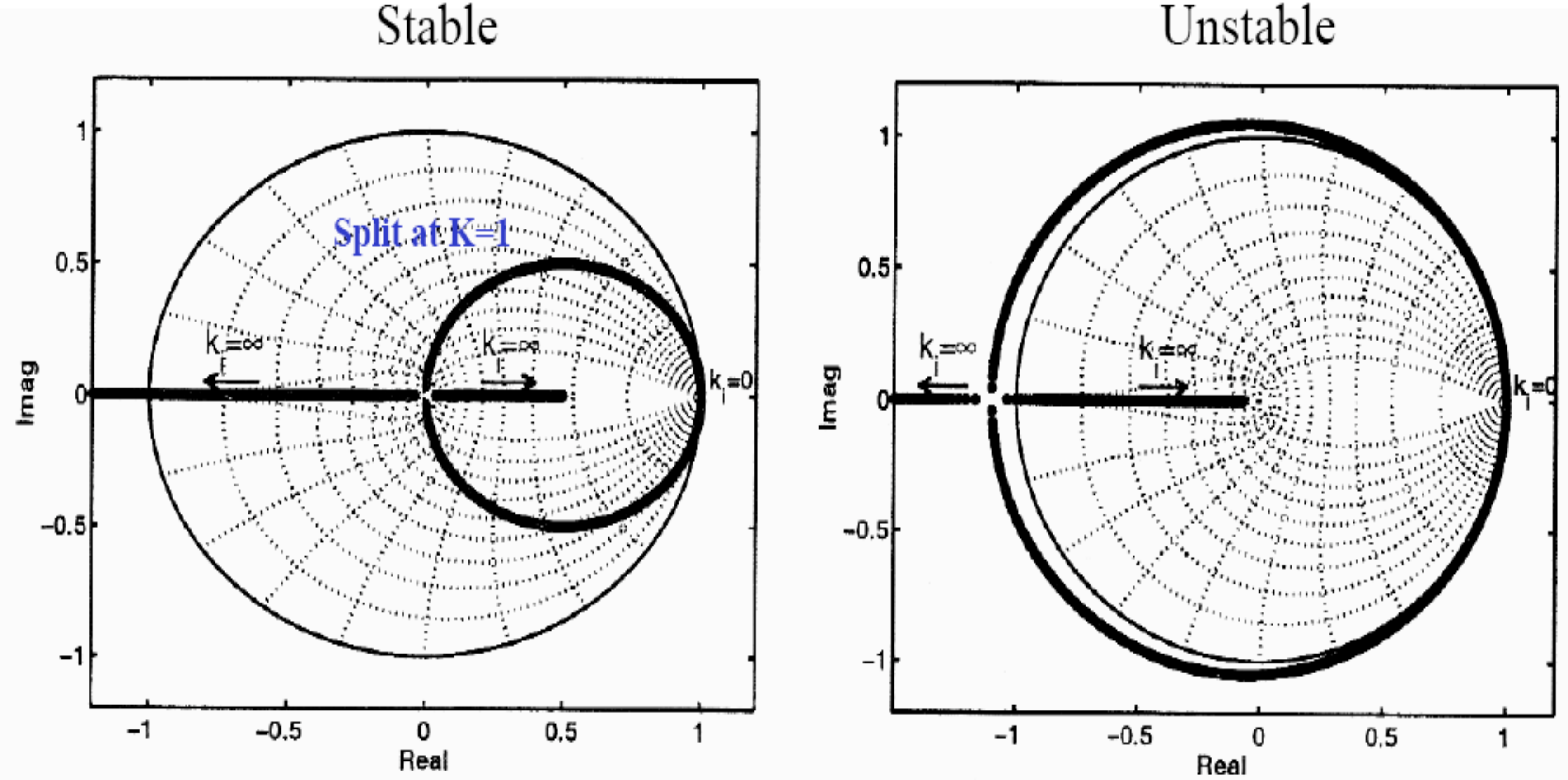
Stability of 1st order sigma-delta modulator

1st order system is stable.



Stability of 2nd order sigma-delta modulator

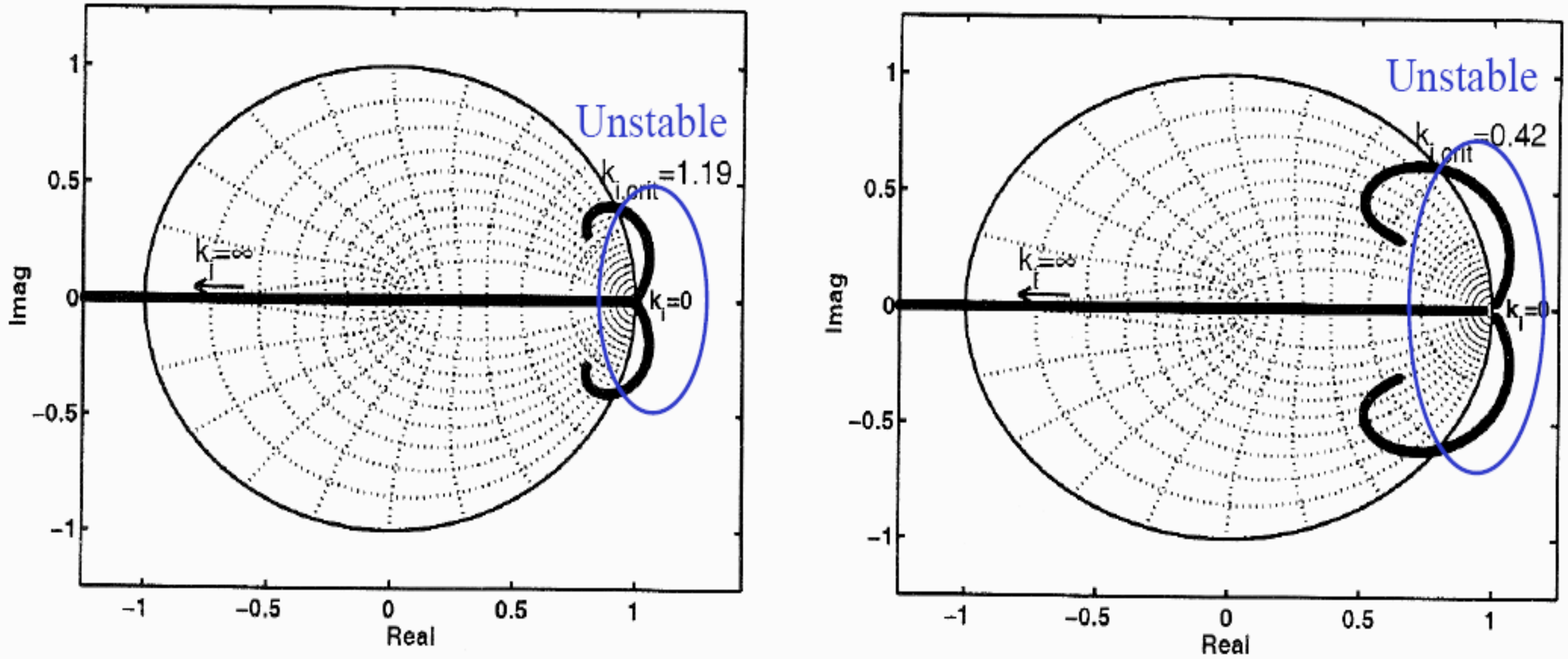
Stable when a_1 is less than unity



(a) Stable loop with a_1=0.5 (b) Unstable loop with a_1=1.05

Stability of 3rd order sigma-delta modulator

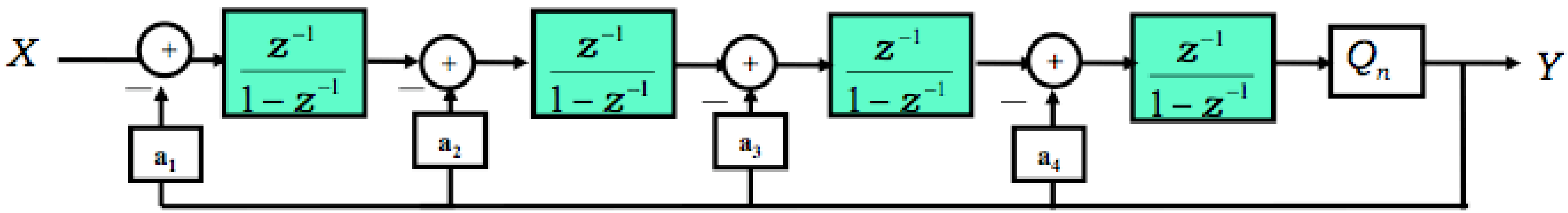
Unstable when k is smaller than the certain value.



(a) Coefficients $(a_1, a_2, a_3) = (0.3, 0.4, 0.5)$ (b) Coefficients $(a_1, a_2, a_3) = (0.3, 0.7, 2)$

Figure 2.20: *Root-locus of third-order $\Delta\Sigma$ converters.*

Actual 4th order Sigma-delta ADC

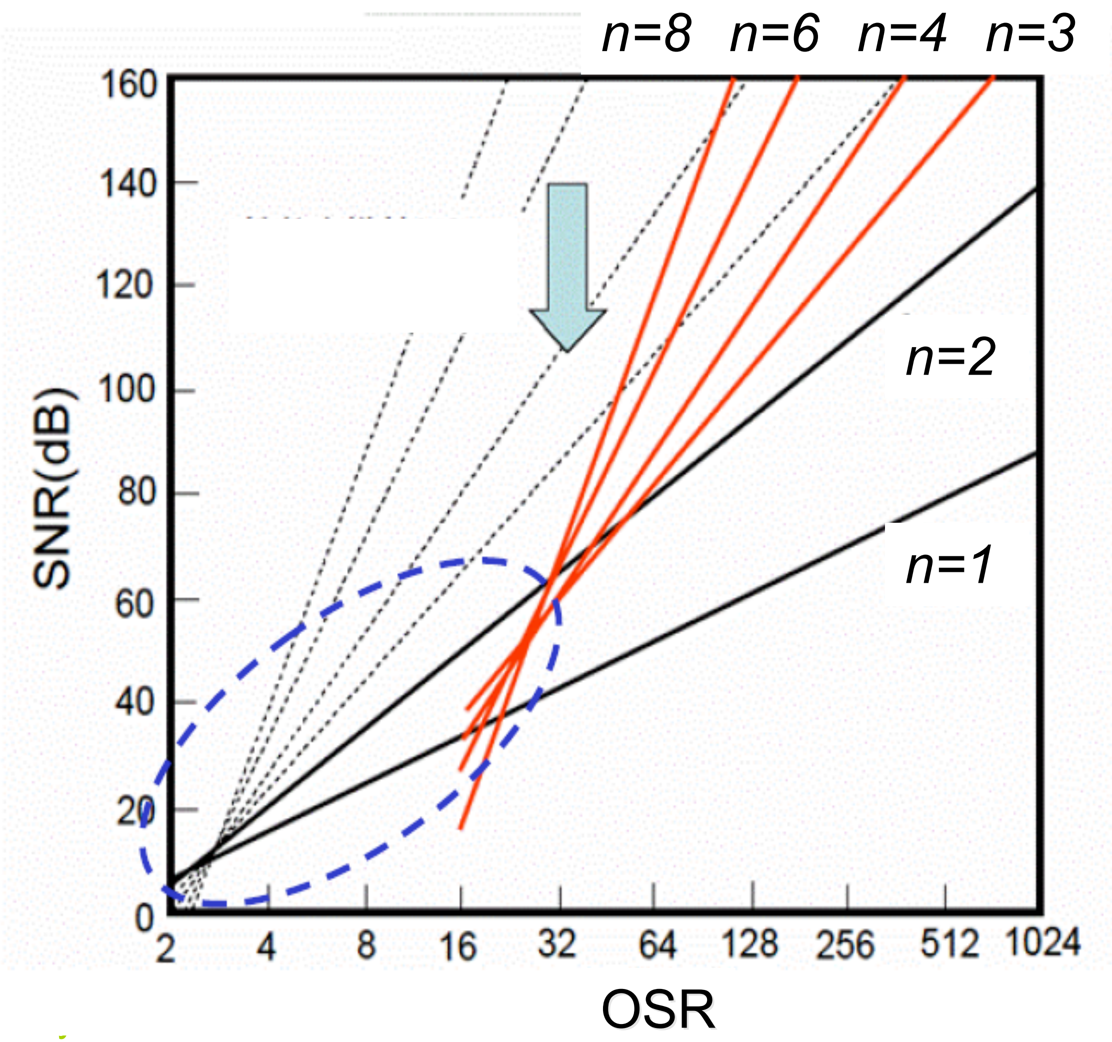


$$NTF: \frac{\left(1-z^{-1}\right)^4}{\left(1-z^{-1}\right)^4 + a_4\left(1-z^{-1}\right)^3 z^{-1} + a_3\left(1-z^{-1}\right)^2 z^{-2} + a_2\left(1-z^{-1}\right)^1 z^{-3} + a_1 z^{-4}}$$

Needs adjustment of coefficients for system stabilization.

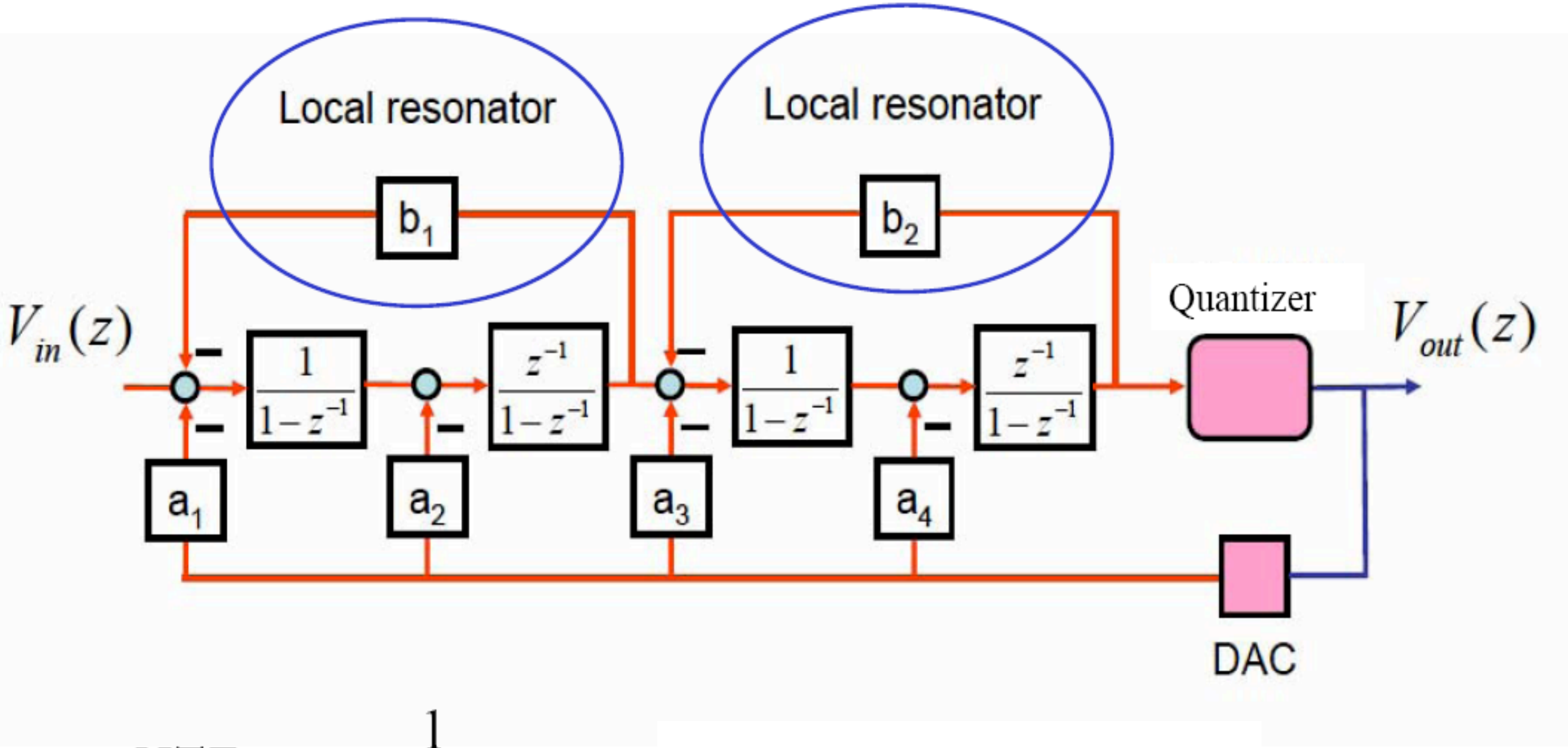
SNR at stable condition

SNR is degraded when the system becomes stable.
In particular, it is heavily degraded at low over sampling rate.



Local resonator

Local resonators are used to make NTF zeroes.

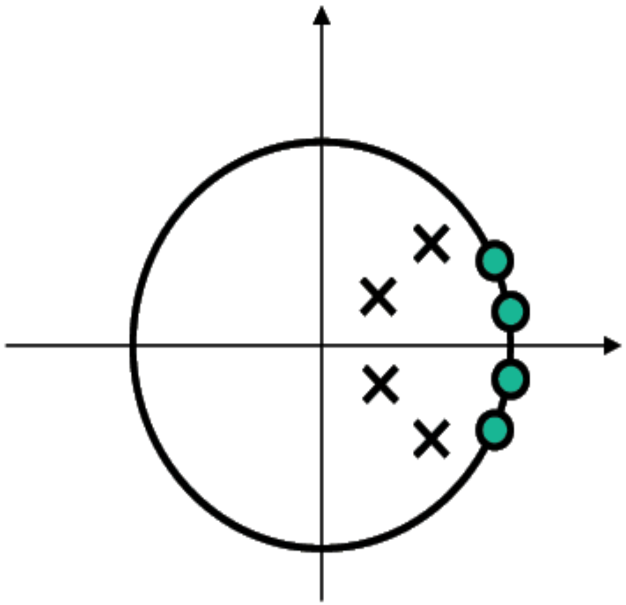


$$NTF : \frac{1}{1 + H(z)F(z)}$$

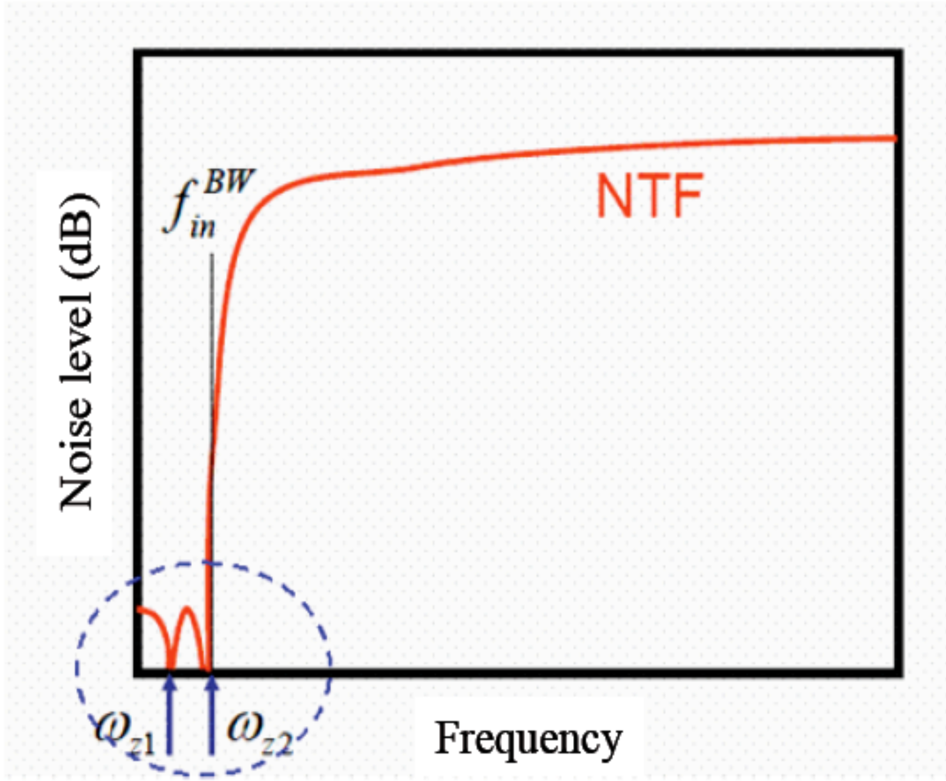
Zero distribution

Distribute the zeroes can reduce in-band noise.

Distribute the zeroes on the $|z| = 1$

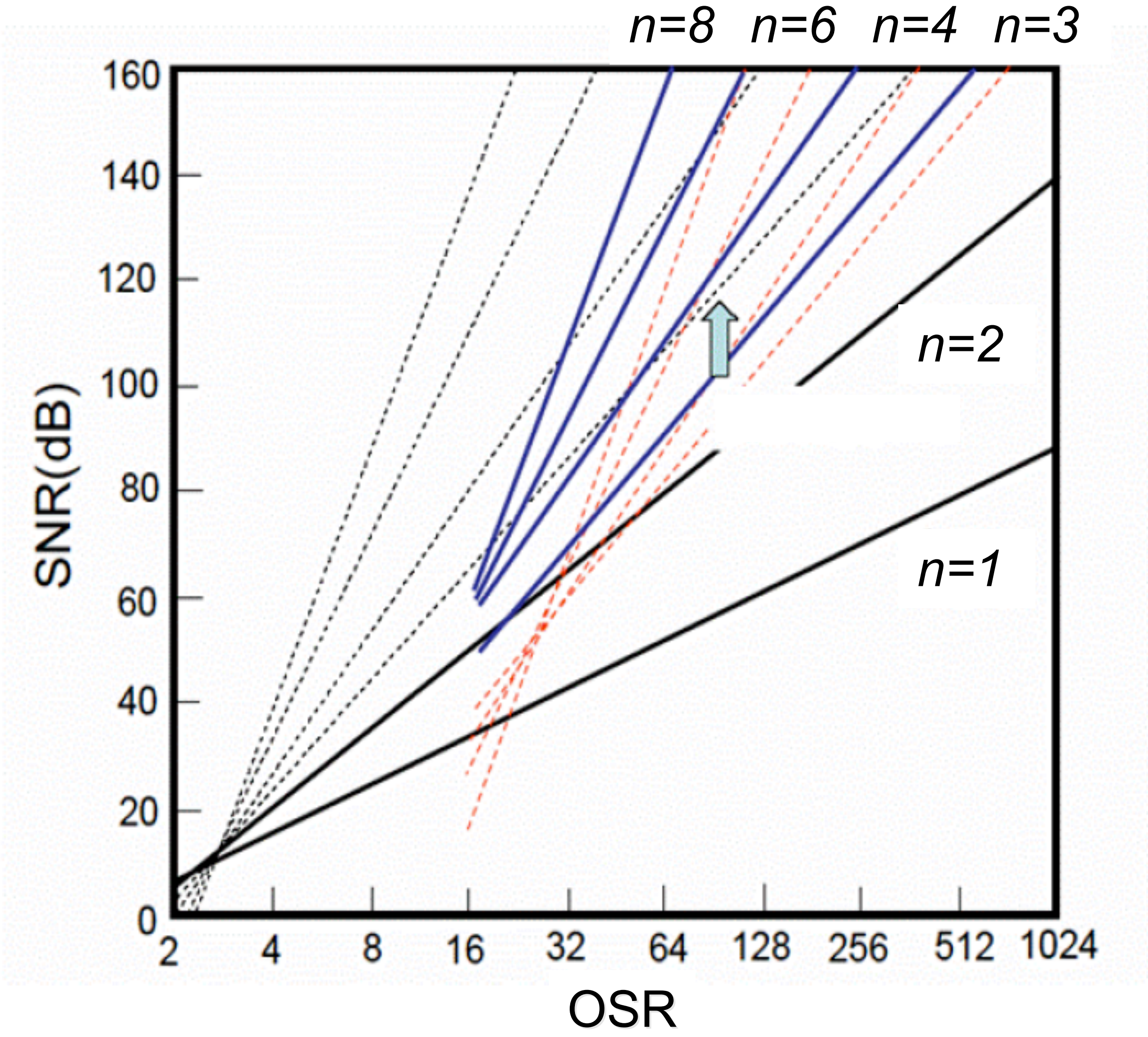


Poles can't be changed so much because of stability.



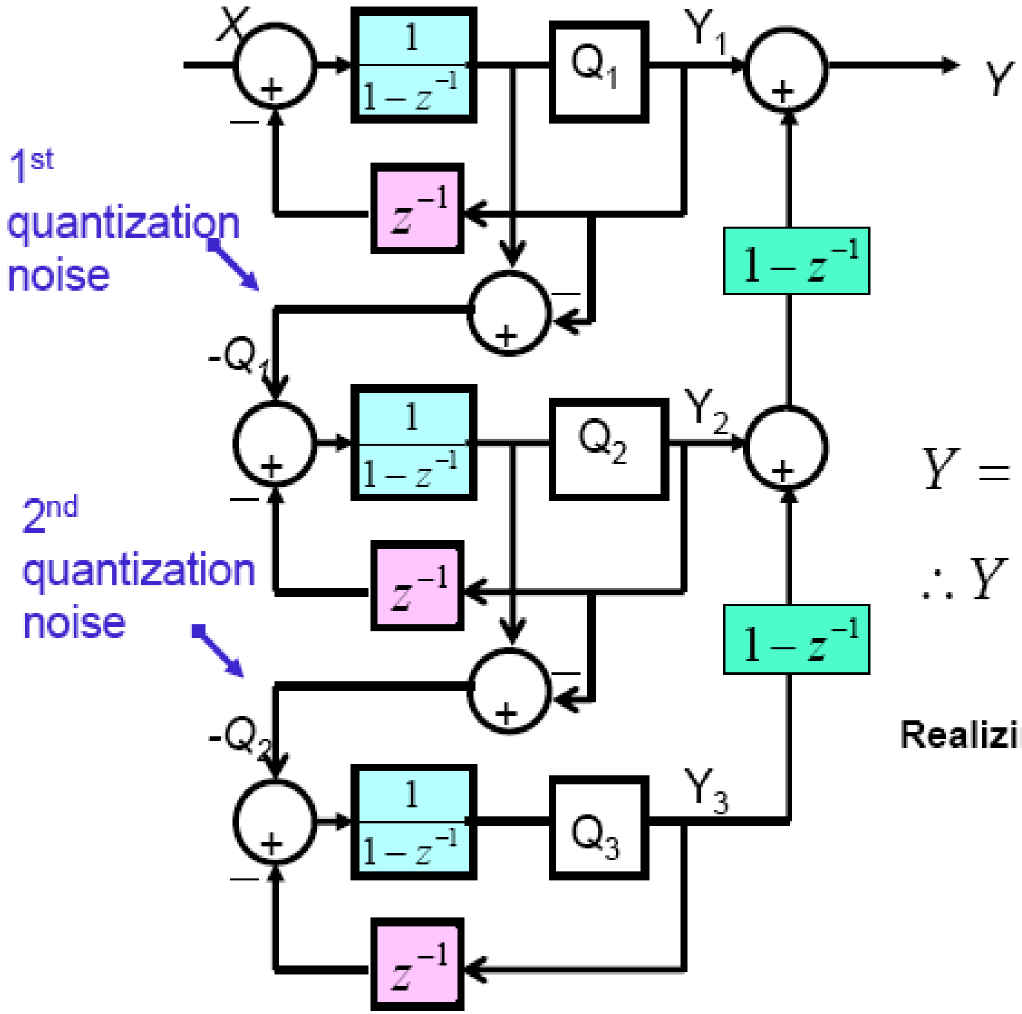
Deep notches in pass band

Effect of zero distribution



MASH (Multi-stage noise shaping)

Feed forwarded multi-stage noise shaping architecture is free from instability, however requires good matching.



$$Y_1 = X + \left(1 - Z^{-1}\right)Q_1$$

$$Y_2 = -Q_1 + \left(1 - Z^{-1}\right)Q_2$$

$$Y_3 = -Q_2 + \left(1 - Z^{-1}\right)Q_3$$

$$Y = Y_1 + \left(1 - Z^{-1}\right)Y_2 + \left(1 - Z^{-1}\right)^2 Y_3$$

$$\therefore Y = X + \left(1 - Z^{-1}\right)^3 Q_3$$

Realizing the stable 3rd order sigma delta modulation.