

Lecture I

- Baseband Pulse Transmission
- Digital Passband Transmission
- Circuit Non-idealities Effect

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- Baseband Pulse Transmission
- Digital Passband Transmission
- Circuit Non-idealities Effect

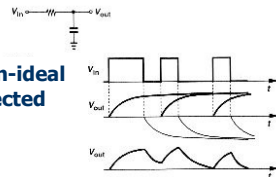
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Digital Baseband Transmission

Major sources of errors in the detection of transmitted digital data:

ISI : InterSymbol Interference

The result of data transmission over a non-ideal channel is that each received pulse is affected by adjacent pulses.



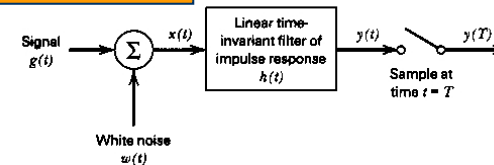
Channel Noise

Detecting a pulse transmitted over a channel that is corrupted by additive noise.



Matched Filter

Linear Receiver Model



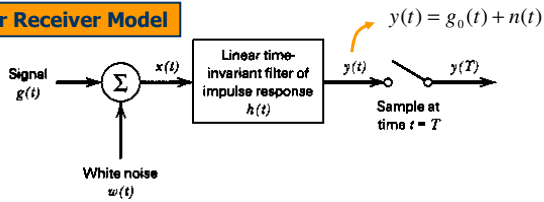
- $g(t)$: transmitted pulse signal, binary symbol '1' or '0'.
- $w(t)$: channel noise, sample function of a white noise process of zero mean and power spectral density $N_0/2$.

$$x(t) = g(t) + w(t) \quad , \quad 0 \leq t \leq T \quad \longrightarrow \quad h(t) \quad \longrightarrow \quad y(t) = g_0(t) + n(t)$$

- Filter Requirements, $h(t)$:
 - Make the instantaneous power in the output signal $g_0(t)$, measured at time $t=T$, as large as possible compared with the average power of the output noise, $n(t)$.

Maximize Signal-to-Noise Ratio

Linear Receiver Model



$$SNR = \frac{\text{instantaneous power in the output signal}}{\text{average output noise power}}$$

$$SNR = \frac{|g_0(T)|^2}{n^2(t)}$$

Objective :

➔ Specify the impulse response $h(t)$ of the filter such that the output signal-to-noise ratio is maximized.

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Math Review

Fourier Transform

$$G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt$$

Inverse Fourier Transform

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f t} df$$

Power Spectral Density of a Random Process $S_x(f)$ applied to a Linear System $H(f)$

$$S_Y(f) = |H(f)|^2 S_X(f)$$

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Compute Signal-to-Noise Ratio

Signal Power

$$g_0(t) = \int_{-\infty}^{\infty} H(f) G(f) e^{j2\pi f t} df$$

$$|g_0(T)|^2 = \left| \int_{-\infty}^{\infty} H(f) G(f) e^{j2\pi f T} df \right|^2$$

Noise Power

$$S_N(f) = \frac{N_0}{2} |H(f)|^2$$

$$\overline{n^2(t)} = \int_{-\infty}^{\infty} S_N(f) df$$

$$\overline{n^2(t)} = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df$$

Signal-to-Noise Ratio

Optimization Problem:

➔ For a given $G(f)$, find $H(f)$ in order to maximize SNR.

$$SNR = \frac{\left| \int_{-\infty}^{\infty} H(f) G(f) e^{j2\pi f T} df \right|^2}{\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df}$$

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Schwarz's Inequality

• If we have 2 complex functions $\phi_1(x)$ and $\phi_2(x)$ in the real variable x , satisfying the conditions:

$$\int_{-\infty}^{\infty} |\phi_1(x)|^2 dx < \infty$$

$$\int_{-\infty}^{\infty} |\phi_2(x)|^2 dx < \infty$$

then we may write that:

$$\left| \int_{-\infty}^{\infty} \phi_1(x) \phi_2(x) dx \right|^2 \leq \int_{-\infty}^{\infty} |\phi_1(x)|^2 dx \int_{-\infty}^{\infty} |\phi_2(x)|^2 dx \quad \text{iff} \quad \phi_1(x) = k \phi_2^*(x)$$

where k : arbitrary constant

setting: $\phi_1(x) = H(f)$ and $\phi_2(x) = G(f) e^{j2\pi f T}$

$$\left| \int_{-\infty}^{\infty} H(f) G(f) e^{j2\pi f T} df \right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |G(f)|^2 df$$

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Matched Filter

$$SNR \leq \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df$$



$$SNR_{max} = \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df$$

$$\phi_1(x) = k \phi_2^*(x)$$

$$H_{opt}(f) = k G^*(f) e^{-j2\pi f T}$$

$$h_{opt}(t) = k \int_{-\infty}^{\infty} G^*(f) e^{-j2\pi f (T-t)} df$$

$$h_{opt}(t) = k \int_{-\infty}^{\infty} G(-f) e^{-j2\pi f (T-t)} df$$

• for a real signal $g(t)$ we have $G^*(f) = G(-f)$

$$h_{opt}(t) = k g(T-t)$$

• The impulse response of the optimum filter, except for the scaling factor k , is a time-reversed and delayed version of the input signal $g(t)$

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Properties of Matched Filters

$$h_{opt}(t) = k g(T-t)$$

$$\begin{aligned} G_0(f) &= H_{opt}(f) G(f) \\ &= k G^*(f) G(f) e^{-j2\pi f T} \\ &= k |G(f)|^2 e^{-j2\pi f T} \end{aligned}$$

$$H_{opt}(f) = k G^*(f) e^{-j2\pi f T}$$

• Taking the inverse Fourier transform at $t=T$:

$$g_0(T) = \int_{-\infty}^{\infty} G_0(f) e^{j2\pi f T} df = k \int_{-\infty}^{\infty} |G(f)|^2 df = k E$$

Where E is the energy of the pulse signal $g(t)$

$$\overline{n^2(t)} = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df$$

$$\overline{n^2(t)} = \frac{N_0}{2} k^2 \int_{-\infty}^{\infty} |G(f)|^2 df = k^2 \frac{N_0}{2} E$$

$$SNR_{max} = \frac{k^2 E^2}{k^2 \frac{N_0}{2} E} = \frac{2E}{N_0}$$

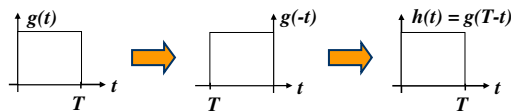
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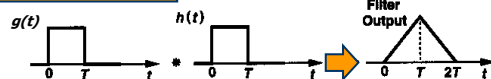
Matched Filter for Rectangular Pulse

$h(t)$ for a rectangular Pulse:

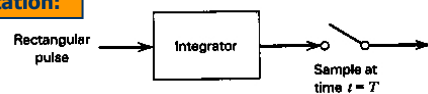
$$h_{opt}(t) = k g(T-t)$$



Filter Output $g(t)*h(t)$:



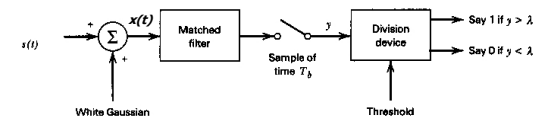
Implementation:



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Error Rate due to Noise



In the interval $0 \leq t \leq T_b$, the received signal:

$$x(t) = \begin{cases} +A + w(t) & \text{symbol '1' was sent} \\ -A + w(t) & \text{symbol '0' was sent} \end{cases}$$

T_b is the bit duration, A is the transmitted pulse amplitude

• The receiver has prior knowledge of the pulse shape but not its polarity.

• There are two possible kinds of error to be considered:
 (1) Symbol '1' is chosen when a '0' was transmitted.
 (2) Symbol '0' is chosen when a '1' was transmitted.

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Error Rate due to Noise

Suppose that symbol '0' was sent: $x(t) = -A + w(t)$, $0 \leq t \leq T_b$

The matched filter output is: $Y = \frac{1}{T_b} \int_0^{T_b} x(t) dt = -A + \frac{1}{T_b} \int_0^{T_b} w(t) dt$

Y is a random variable with Gaussian distribution and a mean of $-A$.

The variance of Y : $\sigma_Y^2 = \overline{(Y + A)^2} = \frac{1}{T_b^2} \int_0^{T_b} \int_0^{T_b} R_w(t, u) dt du$

Where $R_w(t, u)$ is the autocorrelation function of the white noise $w(t)$. Since $w(t)$ is white with a PSD of $N_0/2$:

$$R_w(t, u) = \frac{N_0}{2} \delta(t - u)$$

$$\sigma_Y^2 = \frac{N_0}{2T_b}$$

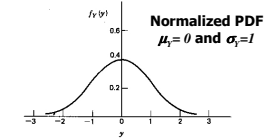
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PDF: Probability Density Function

Gaussian Distribution:

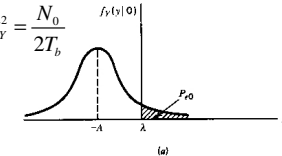
$$f_Y(y) = \frac{1}{\sigma_Y \sqrt{2\pi}} \exp\left[-\frac{(y - \mu_Y)^2}{2\sigma_Y^2}\right]$$



Symbol '0' was sent: $\mu_Y = -A$, $\sigma_Y^2 = \frac{N_0}{2T_b}$

$$P_{e0} = P(y > \lambda | \text{symbol '0' was sent})$$

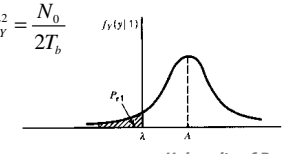
$$= \int_{\lambda}^{\infty} f_Y(y|0) dy$$



Symbol '1' was sent: $\mu_Y = +A$, $\sigma_Y^2 = \frac{N_0}{2T_b}$

$$P_{e1} = P(y < \lambda | \text{symbol '1' was sent})$$

$$= \int_{-\infty}^{\lambda} f_Y(y|1) dy$$



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BER in a PCM receiver

$$P_{e0} = \frac{1}{\sqrt{\pi N_0/T_b}} \int_{\lambda}^{\infty} \exp\left[-\frac{(y + A)^2}{N_0/T_b}\right] dy$$

let $\lambda=0$ and the probabilities of binary symbols: $p_0 = p_1 = 1/2$.

$$z = \frac{y + A}{\sqrt{N_0/T_b}} \quad P_{e0} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp[-z^2] dz$$

where $E_b = A^2 T_b$, is the transmitted signal energy per bit.

the complementary error function: $erfc(u) = \frac{1}{\sqrt{\pi}} \int_u^{\infty} \exp[-z^2] dz$

$$P_{e1} = P_{e0} = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$

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BER in a PCM receiver

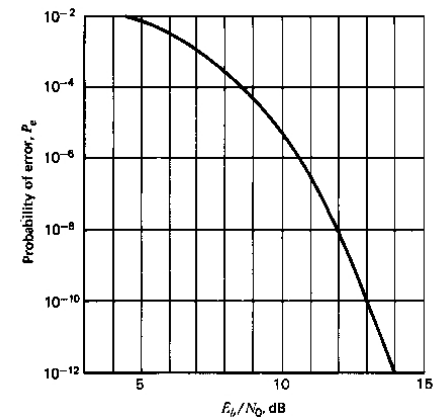
$$P_e = p_0 P_{e0} + p_1 P_{e1}$$

$$P_{e0} = P_{e1}$$

$$p_0 = p_1 = \frac{1}{2}$$

$$P_e = P_{e0} = P_{e1}$$

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$$



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CIRF
Circuit Intégré Radio Fréquence

Lecture I

- **Introduction**
- **Baseband Pulse Transmission**
- **Digital Bandpass Transmission**
- **Circuit Non-idealities Effect**

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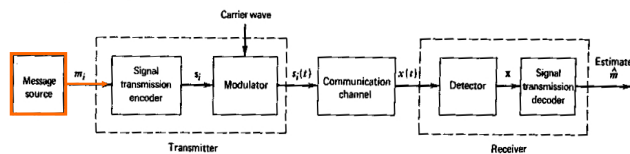
Why Modulation?

- In wired systems, coaxial lines exhibit superior shielding at higher frequencies
- In wireless systems, the antenna size should be a significant fraction of the wavelength to achieve a reasonable gain.
- Communication must occur in a certain part of the spectrum because of FCC regulations.
- Modulation allows simpler detection at the receive end in the presence of non-idealities in the communication channel.

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Message Source



- m_i : one symbol every T seconds
- Symbols belong to an alphabet of M symbols: m_1, m_2, \dots, m_M

- Message output probability:

$$P(m_1) = P(m_2) = \dots = P(m_M)$$

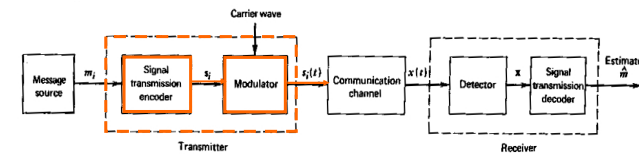
$$p_i = P(m_i) = \frac{1}{M}$$

- Example: Quaternary PCM, 4 symbols: 00, 01, 10, 11

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Transmitter



- **Signal Transmission Encoder**: produces a vector s_i made up of N real elements, where $N \leq M$.
- **Modulator**: constructs a distinct signal $s_i(t)$ representing m_i of duration T .

- Energy of $s_i(t)$:

$$E_i = \int_0^T s_i^2(t) dt, \quad i = 1, 2, \dots, M$$

- $s_i(t)$ is real valued and transmitted every T seconds.

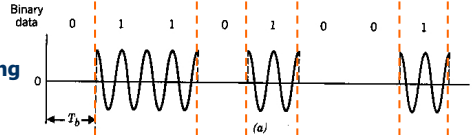
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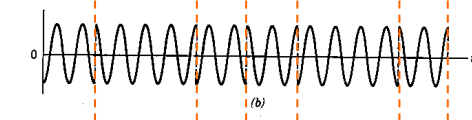
Examples of Transmitted signals: $s_i(t)$

- The modulator performs a step change in the amplitude, phase or frequency of the sinusoidal carrier

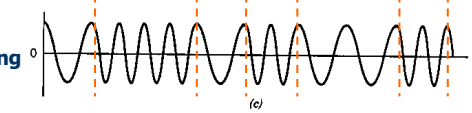
- ASK:**
Amplitude Shift Keying



- PSK:**
Phase Shift Keying



- FSK:**
Frequency Shift Keying

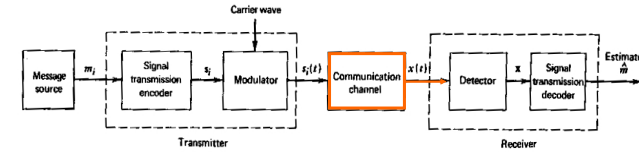


Special case: Symbol Duration $T =$ Bit Duration, T_b

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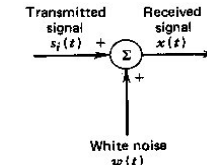
Communication Channel



- Two Assumptions:**
 - The channel is linear (no distortion).
 - $s_i(t)$ is perturbed by an Additive, zero-mean, stationary, White, Gaussian Noise process (AWGN).

- Received signal $x(t)$:

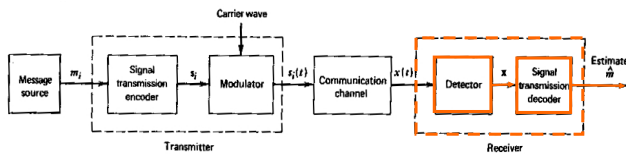
$$x(t) = s_i(t) + w(t), \quad \begin{cases} 0 \leq t \leq T \\ i = 1, 2, \dots, M \end{cases}$$



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Receiver



- TASK:** observe received signal, $x(t)$, for a duration T and make a best estimate of transmitted symbol, m_i .

- Detector:** produces observation vector x .
- Signal Transmission Decoder:** estimates \hat{m} using x , the modulation format and $P(m_i)$.

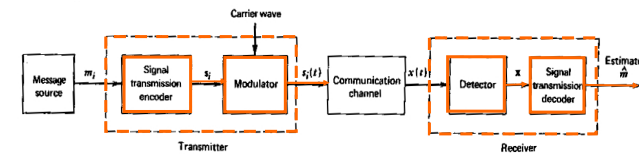
- The requirement is to design a receiver so as to minimize the average probability of symbol error:

$$P_e = \sum_{i=1}^M P(\hat{m} \neq m_i) P(m_i)$$

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Coherent and Non-Coherent Detection



- Coherent Detection:**
 - The receiver is time synchronized with the transmitter.
 - The receiver knows the instants of time when the modulator changes state.
 - The receiver is phase-locked to the transmitter.
- Non-Coherent Detection:**
 - No phase synchronism between transmitter and receiver.

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Gram-Schmidt Orthogonalization Procedure

- we represent the given set of real-valued energy signals $s_1(t), s_2(t), \dots, s_M(t)$, each of duration T :

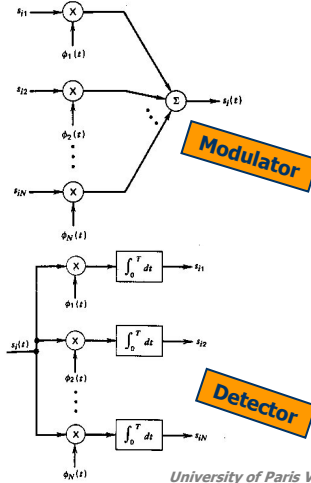
$$s_i(t) = \sum_{j=1}^N s_{ij} \phi_j(t), \quad \begin{cases} 0 \leq t \leq T \\ i = 1, 2, \dots, M \end{cases}$$

- where the coefficients of the expansion are defined by:

$$s_{ij} = \int_0^T s_i(t) \phi_j(t) dt, \quad \begin{cases} i = 1, 2, \dots, M \\ j = 1, 2, \dots, N \end{cases}$$

- the real-valued basis functions $\phi_1(t), \phi_2(t), \dots, \phi_N(t)$ are orthonormal:

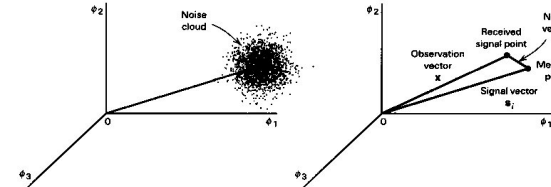
$$\int_0^T \phi_i(t) \phi_j(t) dt = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$



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Coherent Detection of Signals in Noise



- Signal Vector s_i :

$$s_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}, \quad i = 1, 2, \dots, M$$

$$x(t) = s_i(t) + w(t), \quad \begin{cases} 0 \leq t \leq T \\ i = 1, 2, \dots, M \end{cases}$$

- $w(t)$ is a sample function of an AWGN with power spectral density $N_0/2$.

- Observation vector x :

$$x = s_i + w, \quad i = 1, 2, \dots, M$$

where w is the noise vector.

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Coherent Binary PSK:

- $M=2, N=1$ $s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t)$ $s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi)$

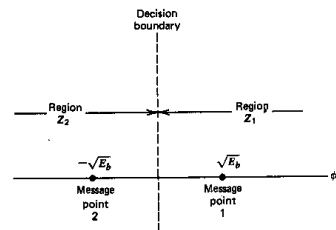
- To ensure that each transmitted bit contains an integral number of cycles of the carrier wave, $f_c = nc/T_b$, for some fixed integer nc .

- One basis function: $\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_b$

- Signal constellation consists of two message points:

$$s_{11} = \int_0^{T_b} s_1(t) \phi_1(t) dt = \sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t) \phi_1(t) dt = -\sqrt{E_b}$$



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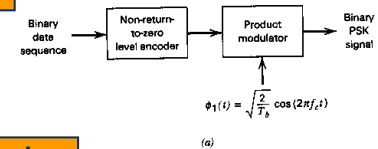
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Generation and Detection of Coherent Binary PSK

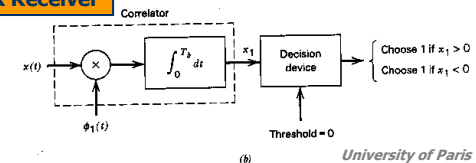
- Assuming white Gaussian Noise with $PSD = N_0/2$, The Bit Error Rate for coherent binary PSK is:

$$P_e = \frac{1}{2} \operatorname{erfc} \left(\sqrt{\frac{E_b}{N_0}} \right)$$

Binary PSK Transmitter



Coherent Binary PSK Receiver



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Coherent QPSK:

- $M=4, N=2$:

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], & 0 \leq t \leq T \\ 0, & \text{elsewhere} \end{cases}$$

$$s_1(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{\pi}{4}\right)$$

$$s_2(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + 3\frac{\pi}{4}\right)$$

$$s_3(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + 5\frac{\pi}{4}\right)$$

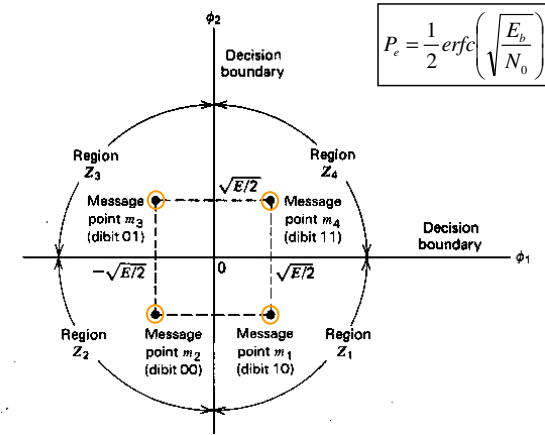
$$s_4(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + 7\frac{\pi}{4}\right)$$

- **Two basis function:** $\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t)$, $0 \leq t \leq T$
 $\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t)$, $0 \leq t \leq T$

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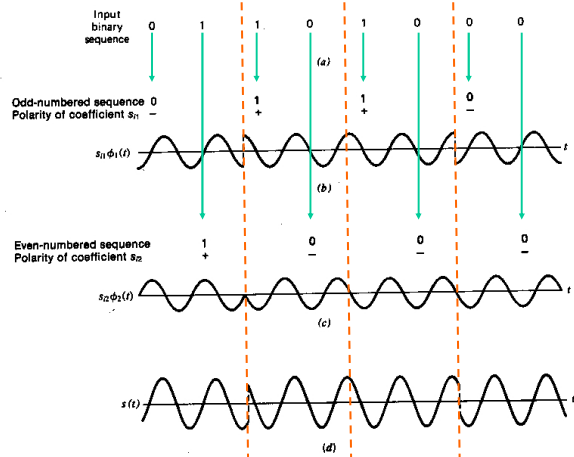
Constellation Diagram of Coherent QPSK System



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QPSK waveform: 01101000

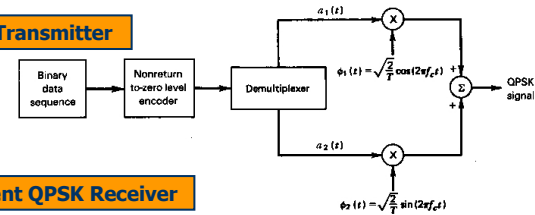


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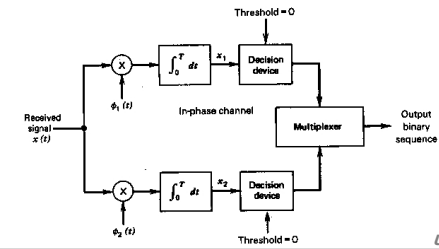
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Generation and Detection of Coherent QPSK Signals

QPSK Transmitter



Coherent QPSK Receiver



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Power Spectra of BPSK, QPSK and M-ary PSK

• Symbol Duration:

$$T = T_b \log_2 M$$

• Power Spectral Density of an M-ary PSK signal:

$$S_B(f) = 2E \operatorname{sinc}^2(Tf) \\ = 2E_b \log_2 M \operatorname{sinc}^2(T_b f \log_2 M)$$

