

“Smart Basket Ball”

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References

- <http://hyperphysics.phy-astr.gsu.edu/hbase/traj.html>


The Motion Equations

These motion equations apply only in the case of constant acceleration.


$$y = \int v dt$$

$$= \int (v_0 + at) dt$$

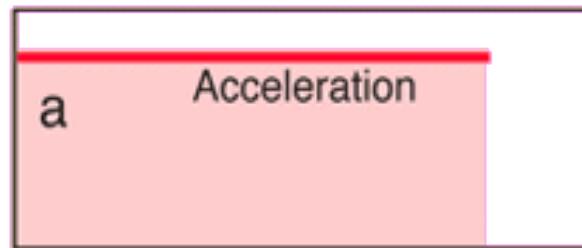
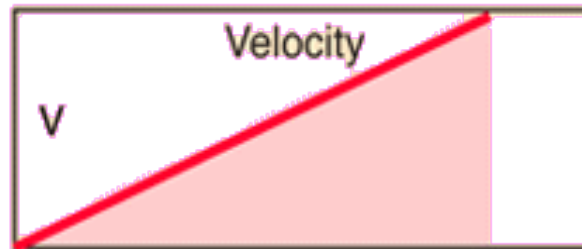
$$y = y_0 + v_0 t + \frac{1}{2} at^2$$

Integrate velocity to get position 

$$v = \int a dt = v_0 + at$$

Integrate acceleration to get velocity 

$$a = \text{constant}$$



time \rightarrow

Motion relationships in one dimension.

$$y = y_0 + v_0 t + \frac{1}{2} at^2$$



Derivative of position is velocity

$$v = \frac{dy}{dt}$$

$$v = v_0 + at$$



Derivative of velocity is acceleration

$$a = \frac{dv}{dt} = a$$

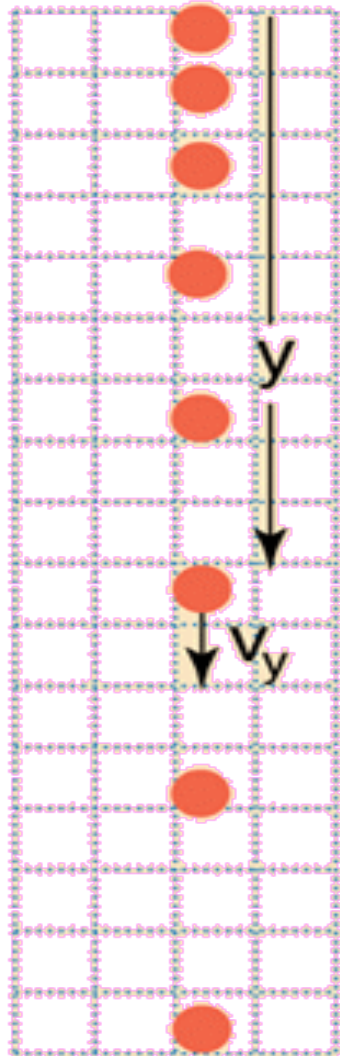
Free Fall

In the absence of frictional drag, an object near the surface of the earth will fall with the constant acceleration of gravity g .

Images of an object in freefall at constant time intervals. Note that the distance traveled in each successive interval is larger.



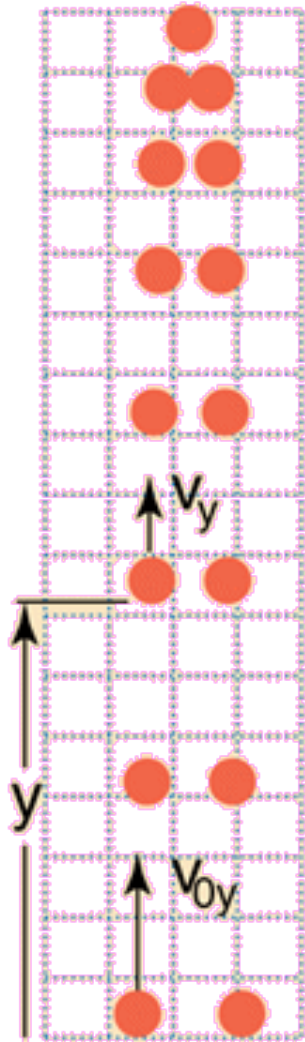
$g = 9.8 \text{ m/s}^2$
so that the velocity increases 9.8 m/s each second.



$$v_y = gt$$
$$y = \frac{1}{2}gt^2$$

Taking $g = 9.8 \text{ m/s}^2$

Vertical Trajectory



$$v_y = v_{0y} - gt$$
$$y = v_{0y} t - \frac{1}{2} g t^2$$

Taking $g = 9.8 \text{ m/s}^2$

Horizontal Launch

Trajectories can be described by the general **motion equations** for constant acceleration. The key idea is that the horizontal and vertical motions can be separated. The motion equations obtained constitute a complete description of the motion, given the initial conditions.

Horizontal Motion →

$$a_x = 0$$

$$v_x = v_{0x}$$

$$x = v_{0x} t$$

Vertical Motion ↓

$$a_y = -g$$

$$v_y = v_{0y} - gt$$

$$y = v_{0y} t - \frac{1}{2}gt^2$$

+↑ Upward chosen as positive direction, so the y values will be negative.

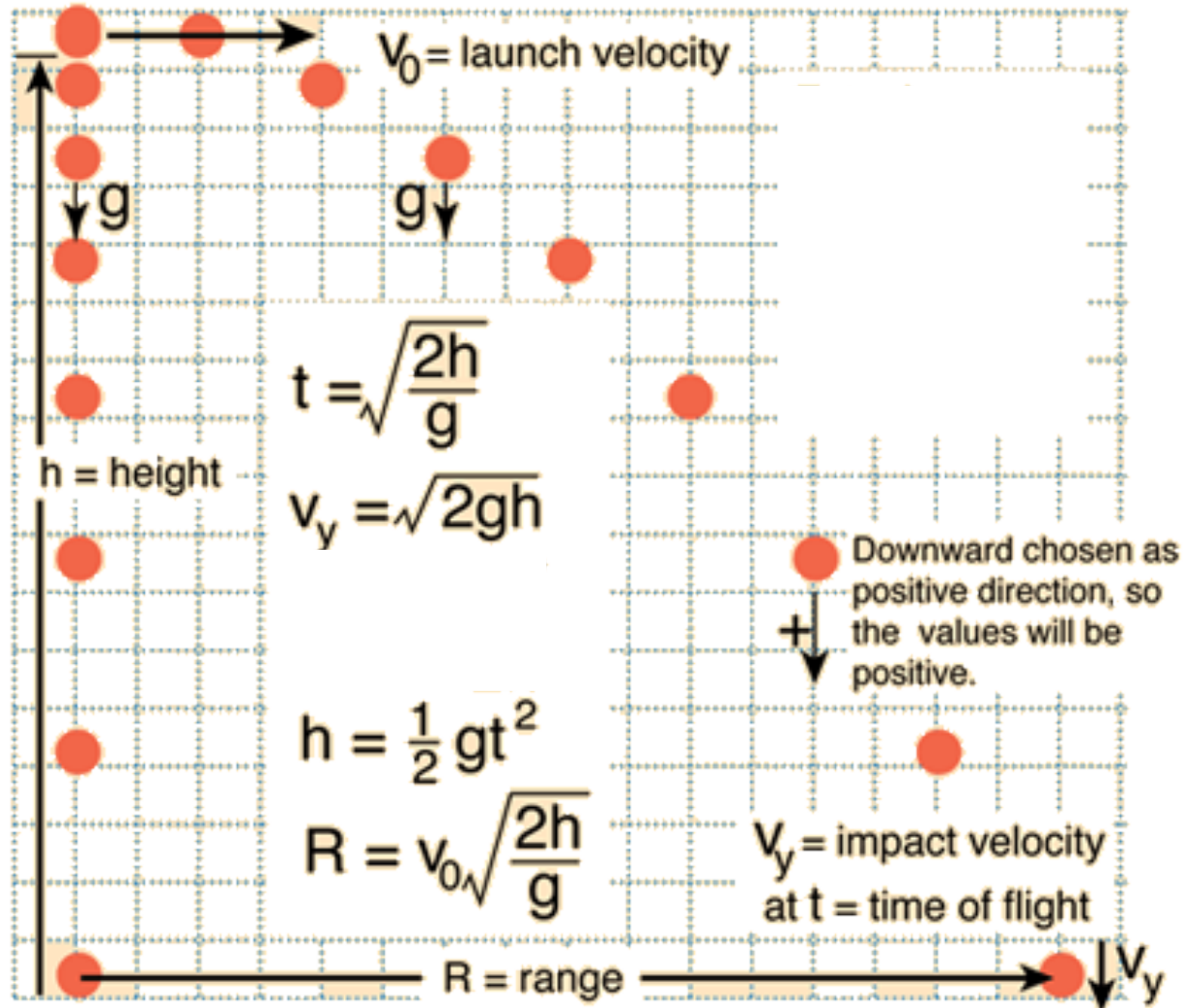
Vertical acceleration is g, regardless of horizontal motion

Successive x intervals are equal, showing zero acceleration.

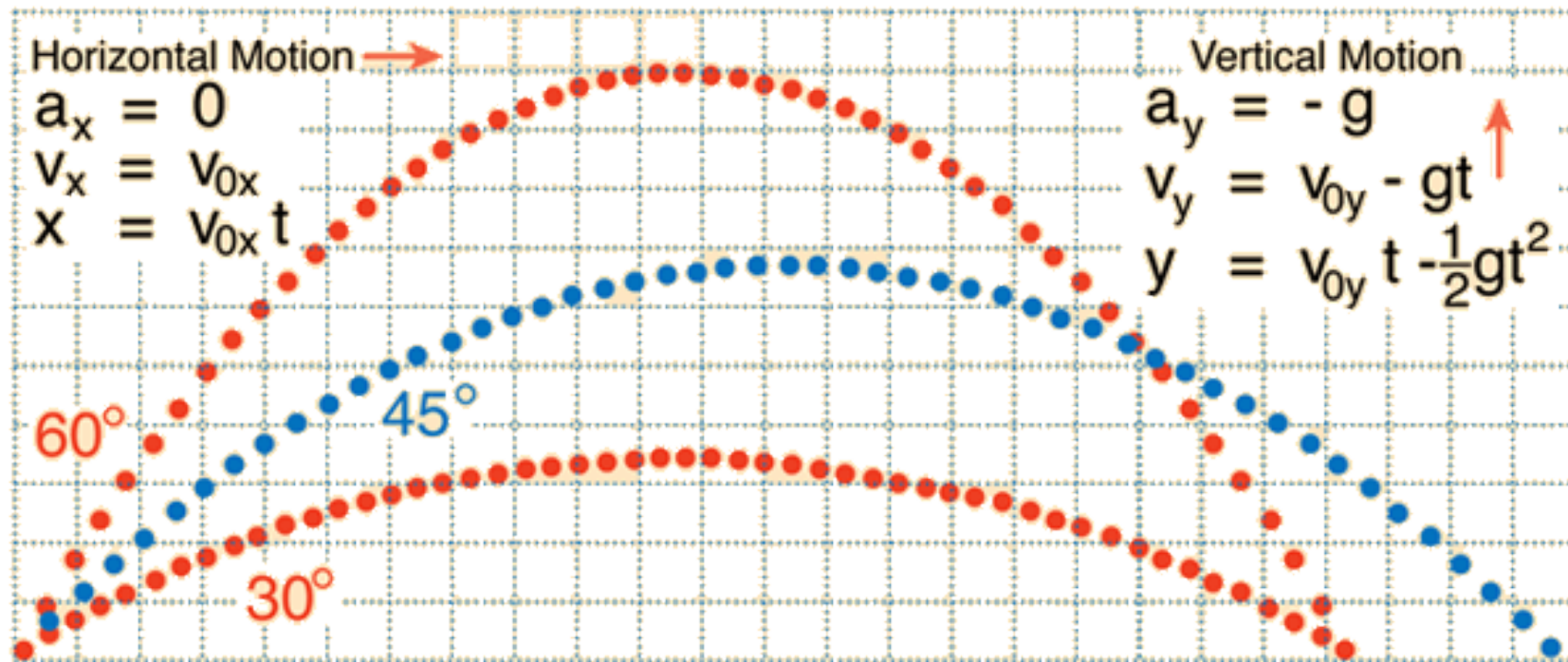
Vertical distance, as well as velocity and acceleration, is independent of x motion.

Horizontal distance is $x = v_{0x} t$ regardless of vertical motion.

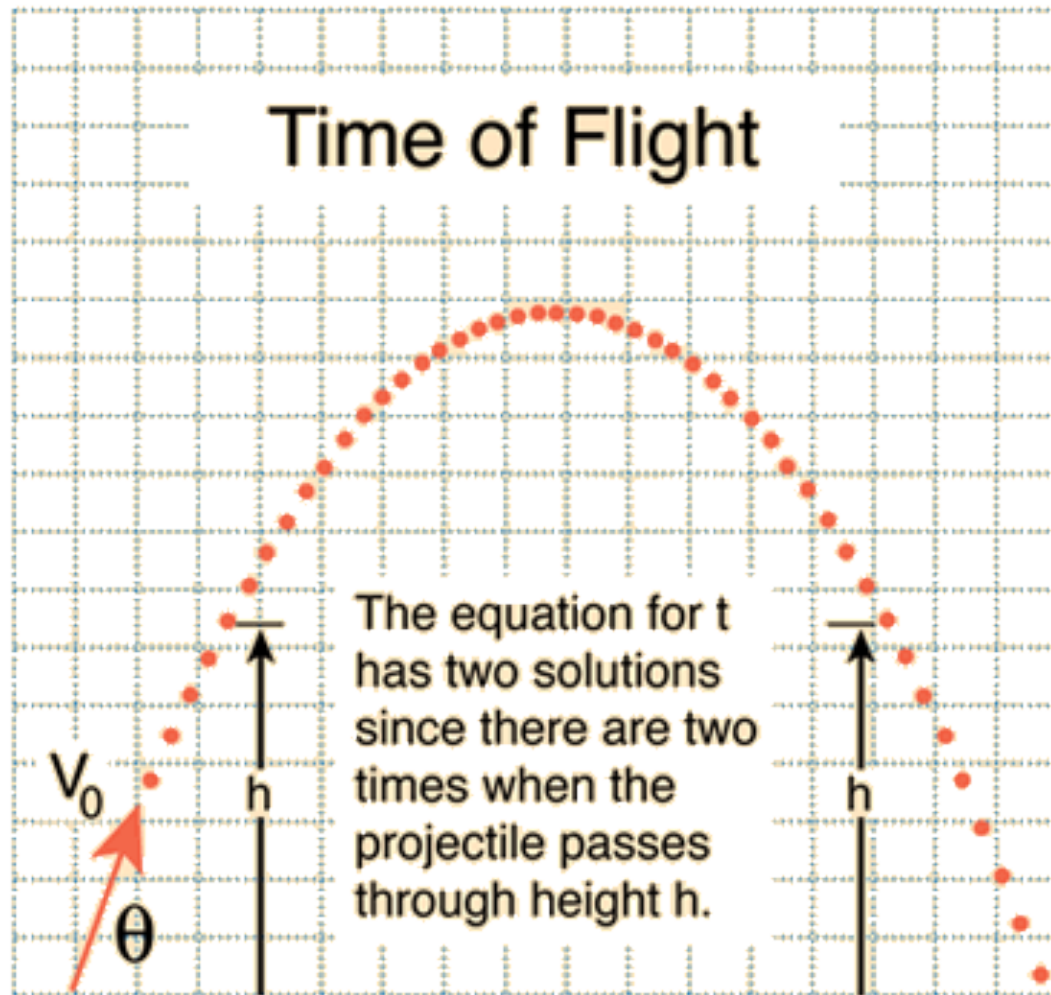
Horizontal Launch



General Ballistic Trajectory



Time of Flight



The basic **motion equation**

$$h = v_{0y} t - \frac{1}{2} g t^2$$

can be used to find the time of flight at height h , giving:*

$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2h}{g}}$$

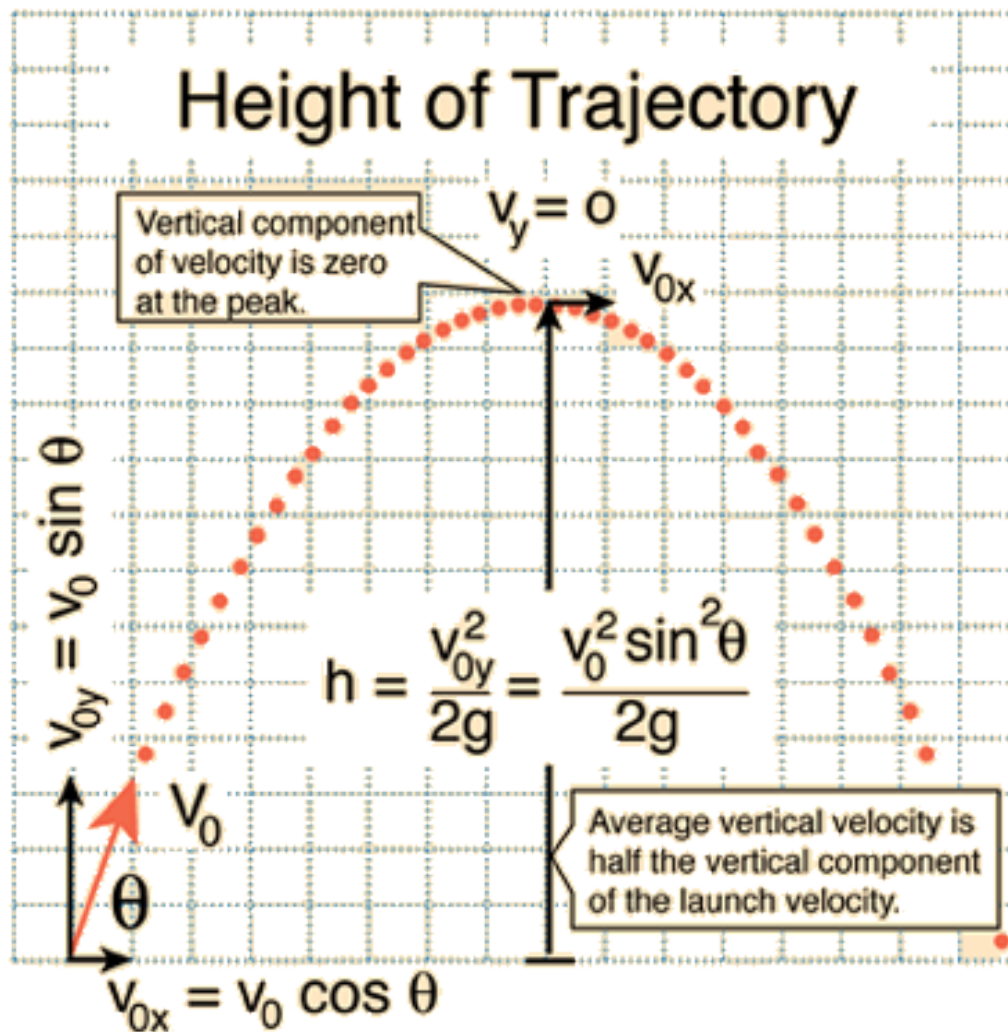
Note that there is no real solution if

$$\frac{2h}{g} > \frac{v_{0y}^2}{g^2} \quad \text{or} \quad h > \frac{v_{0y}^2}{2g}$$

since such values of h are above the peak of the trajectory. For the value $h=0$:

$$t = 0 \quad \text{and} \quad t = \frac{2v_{0y}}{g}$$

Height of Trajectory



The basic **motion equation**

$$y = \bar{v}_y t$$

can be used to find the height.
The average vertical speed is:

$$\bar{v}_y = \frac{v_{0y} + 0}{2} = \frac{v_{0y}}{2}$$

The time at the peak is obtained by solving for the time at zero vertical speed:

$$0 = v_{0y} - gt_{\text{peak}}$$

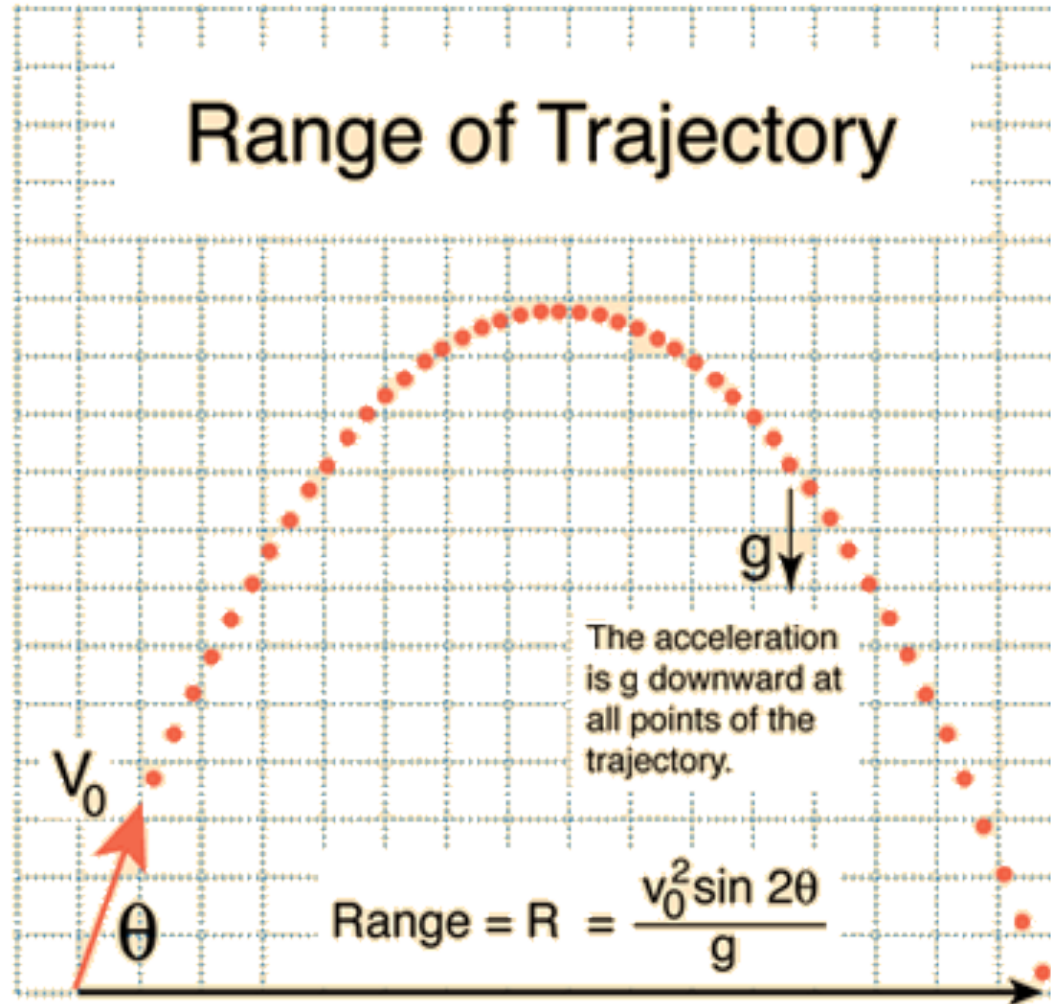
This gives:

$$t_{\text{peak}} = \frac{v_{0y}}{g}$$

and substituting:

$$h = y_{\text{peak}} = \frac{v_{0y}^2}{2g}$$

Range of Trajectory



The basic **motion equation**

$$x = v_{0x} t$$

can be used to find the range. By symmetry, the total **time of flight** is equal to twice the time at the peak:

$$t_{\text{range}} = 2t_{\text{peak}} = \frac{2v_{0y}}{g}$$

This gives:

$$R = \frac{2v_{0x} v_{0y}}{g}$$

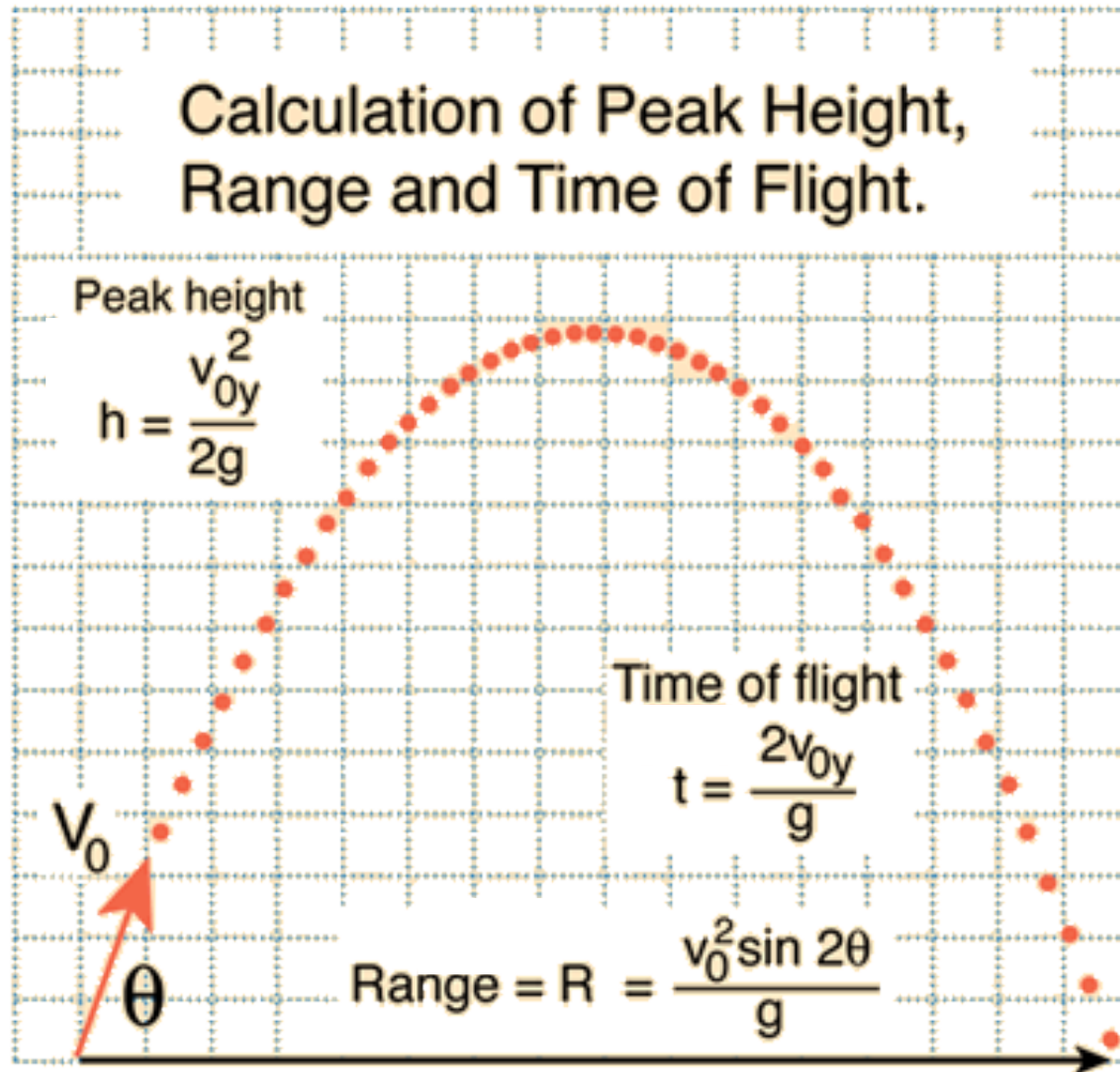
$$R = \frac{2v_0^2 \sin \theta \cos \theta}{g}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

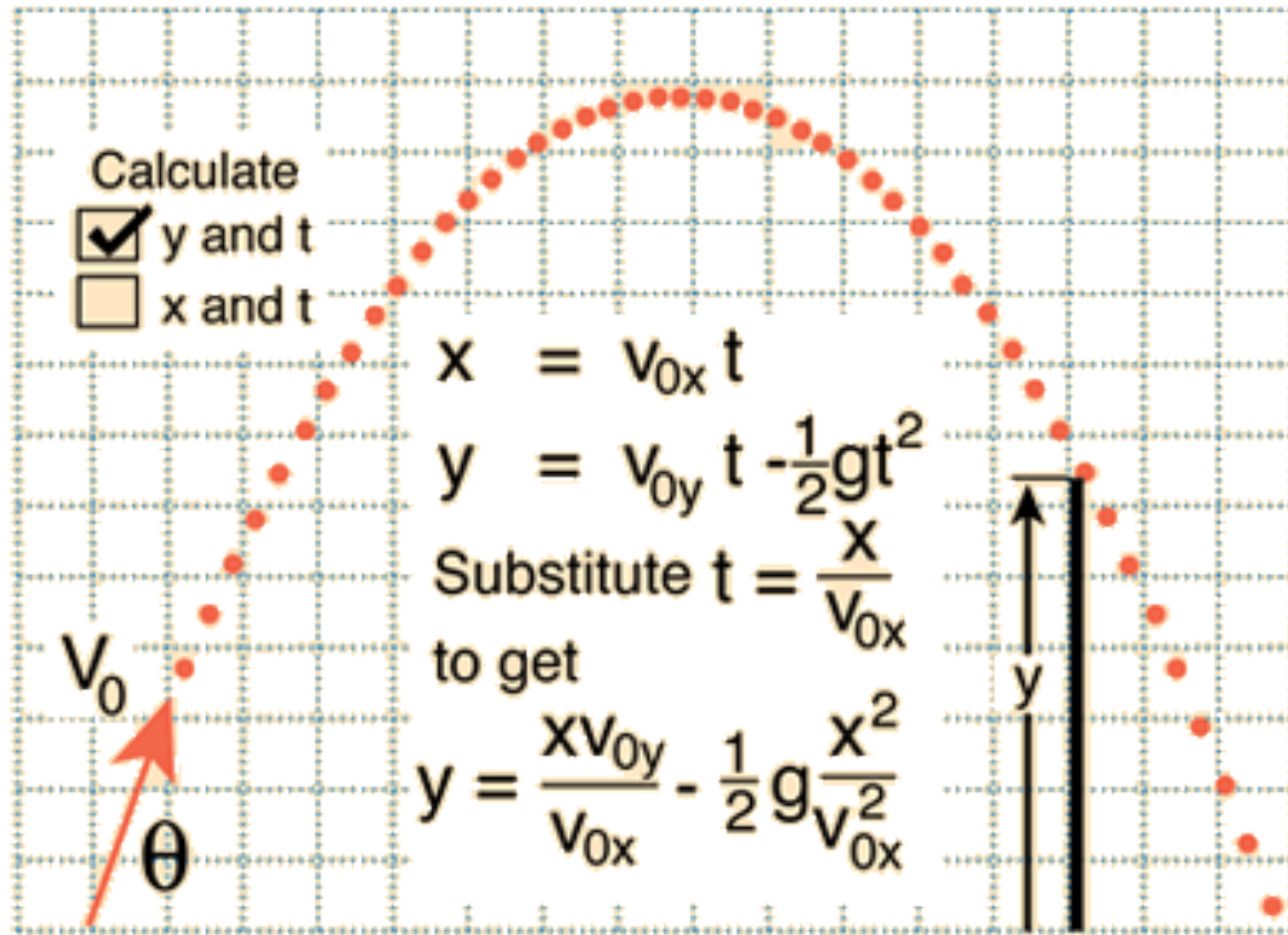
using the **trig identity**:

$$\sin 2\theta = 2 \sin \theta \cos \theta.$$

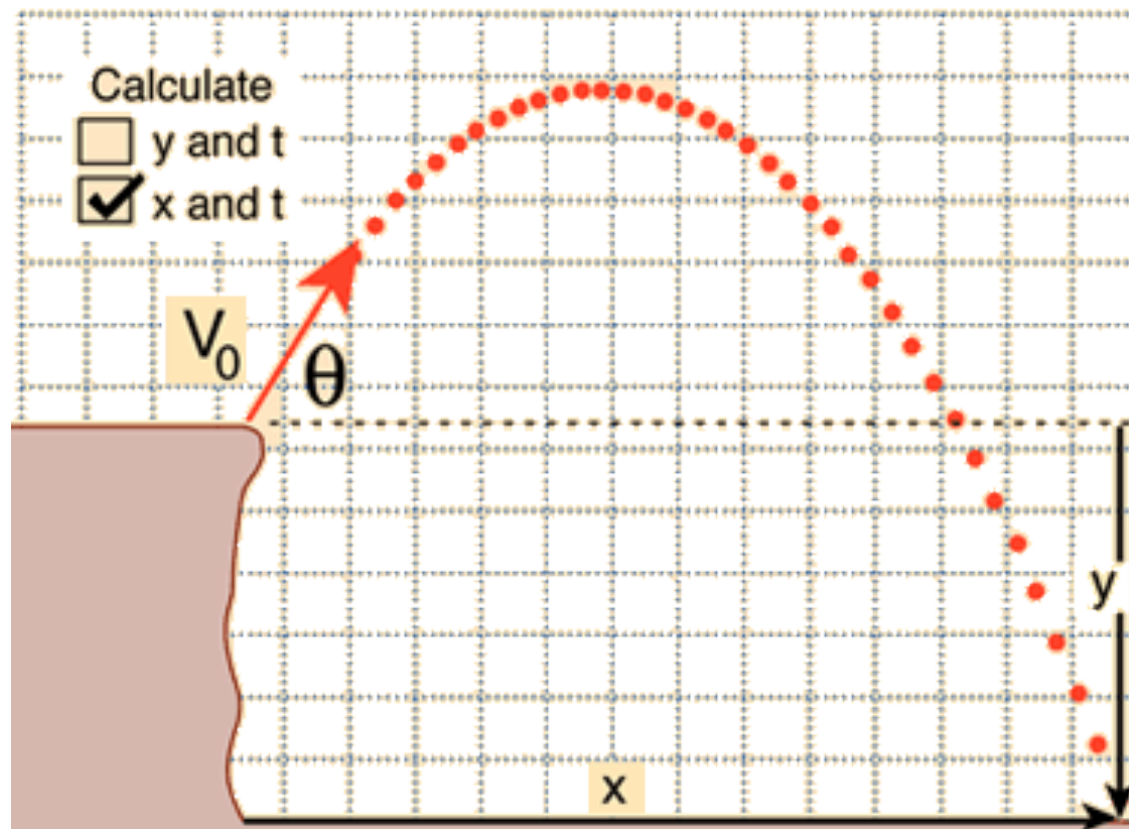
Peak Height, Range and Time of Flight



Express y in terms of x



Where will it land?



$$x = v_{0x} t$$

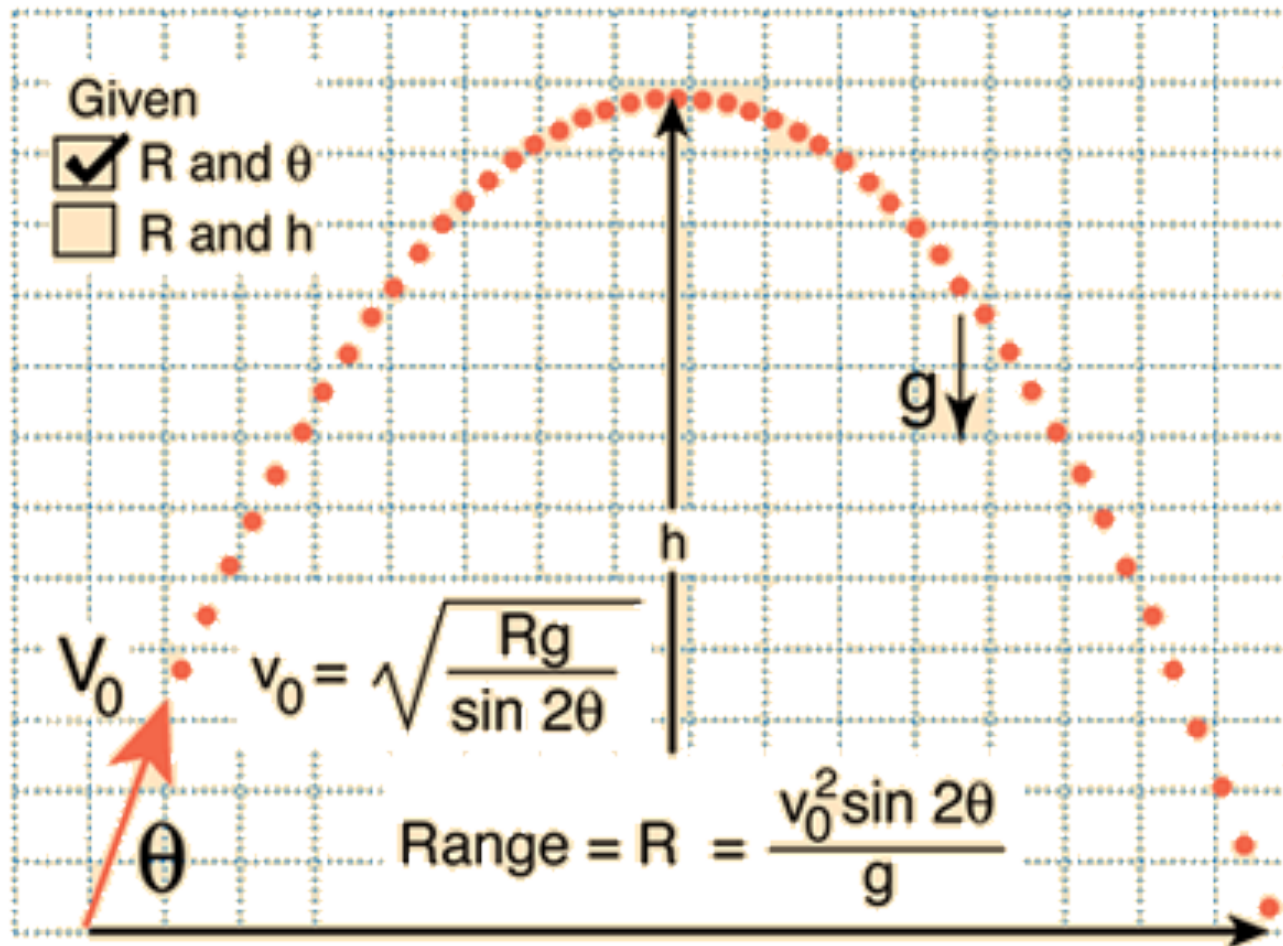
$$y = v_{0y} t - \frac{1}{2}gt^2$$

Using the **quadratic formula** to solve for t gives two values of time for a given value of y:

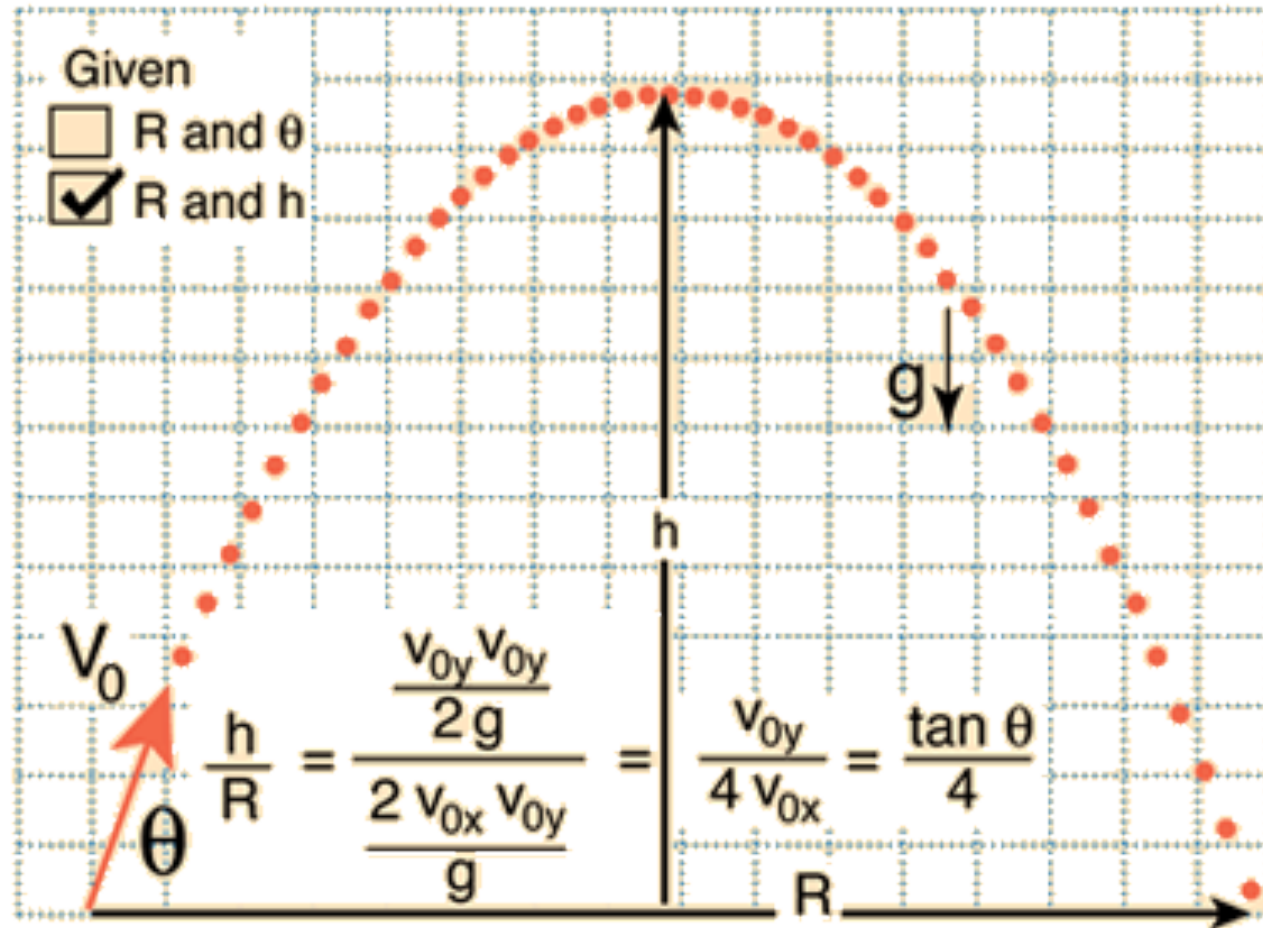
$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2y}{g}}$$

Substitution of the two time values gives the two values of x corresponding to a given height y.

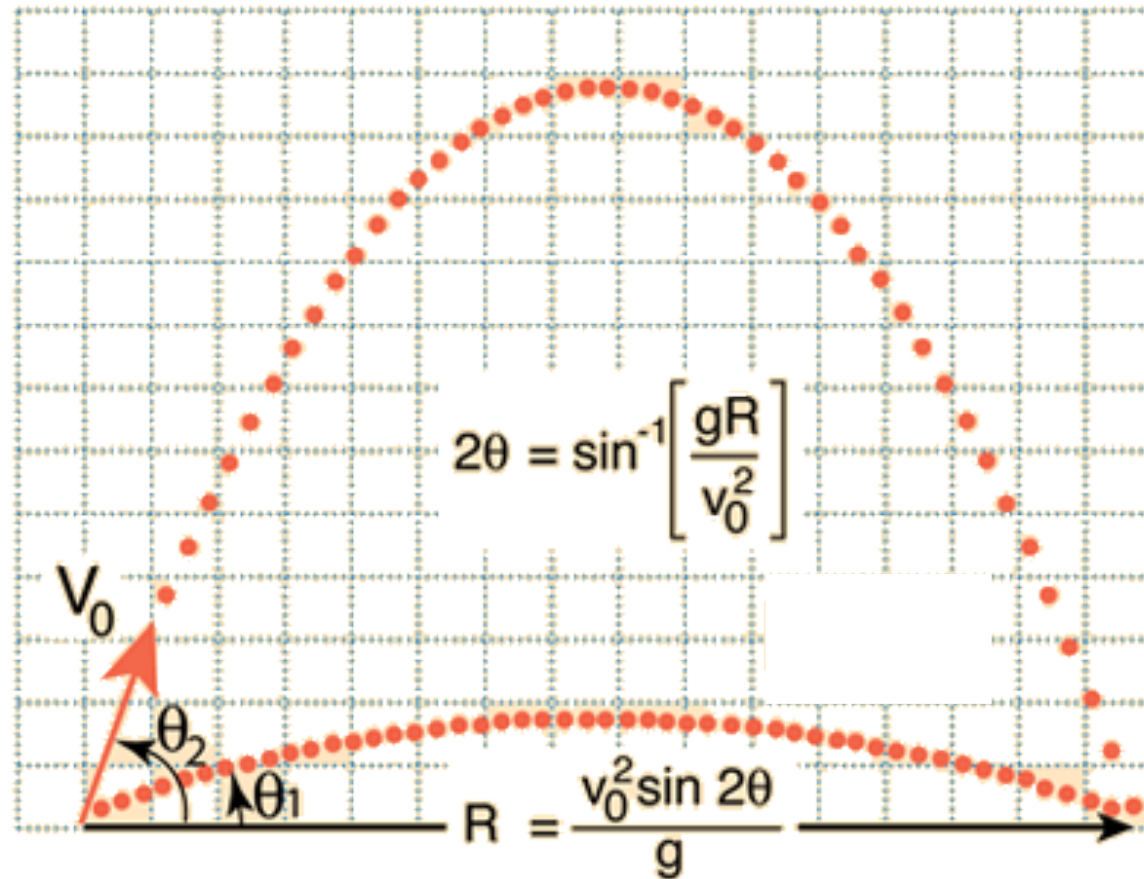
Launch velocity I



Launch Velocity II



Angle of Launch I



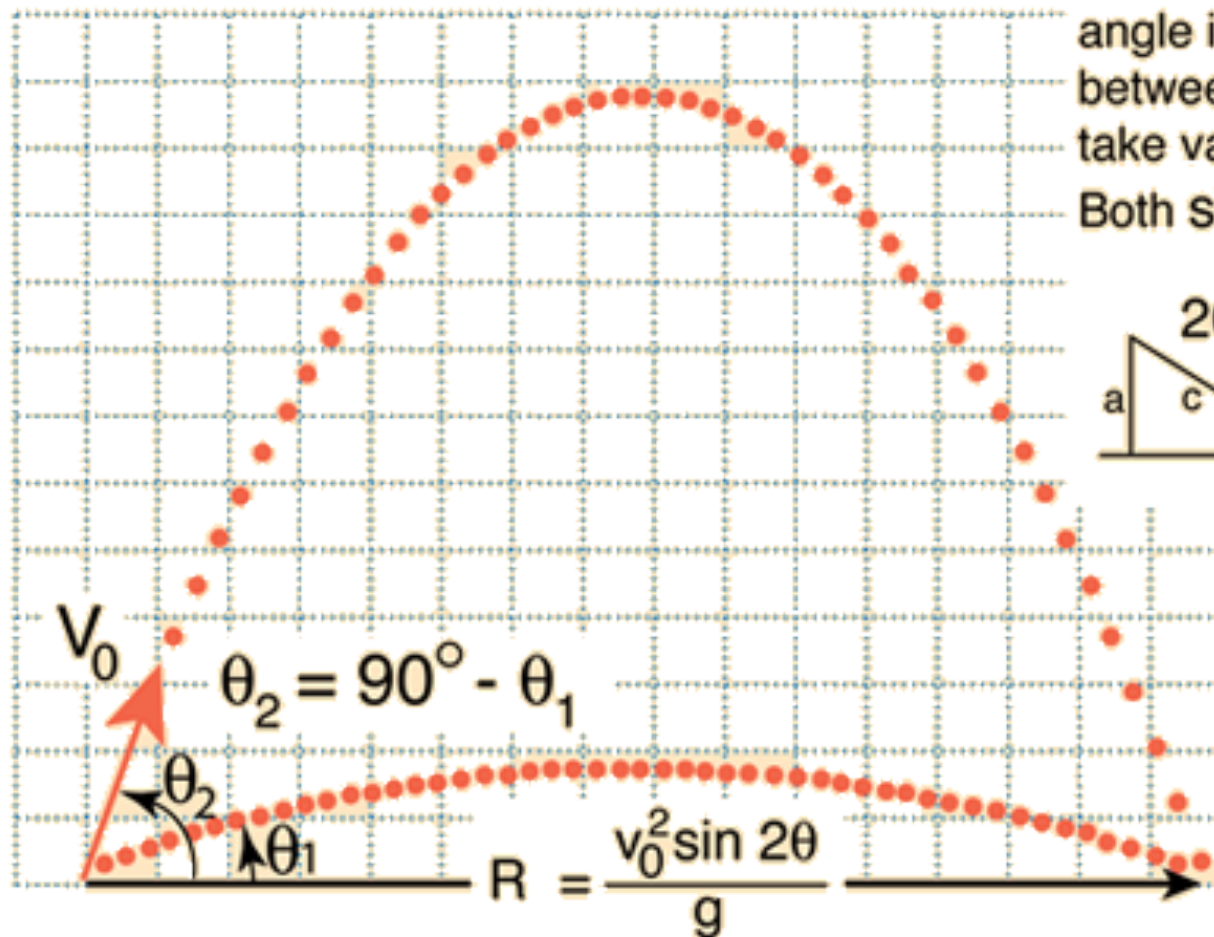
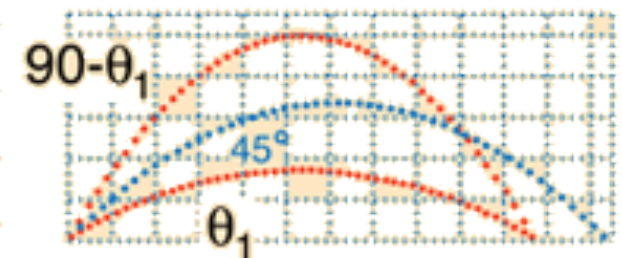
Angle of Launch II

When the **launch angle** is calculated from the relationship $2\theta = \sin^{-1}\left[\frac{gR}{v_0^2}\right]$

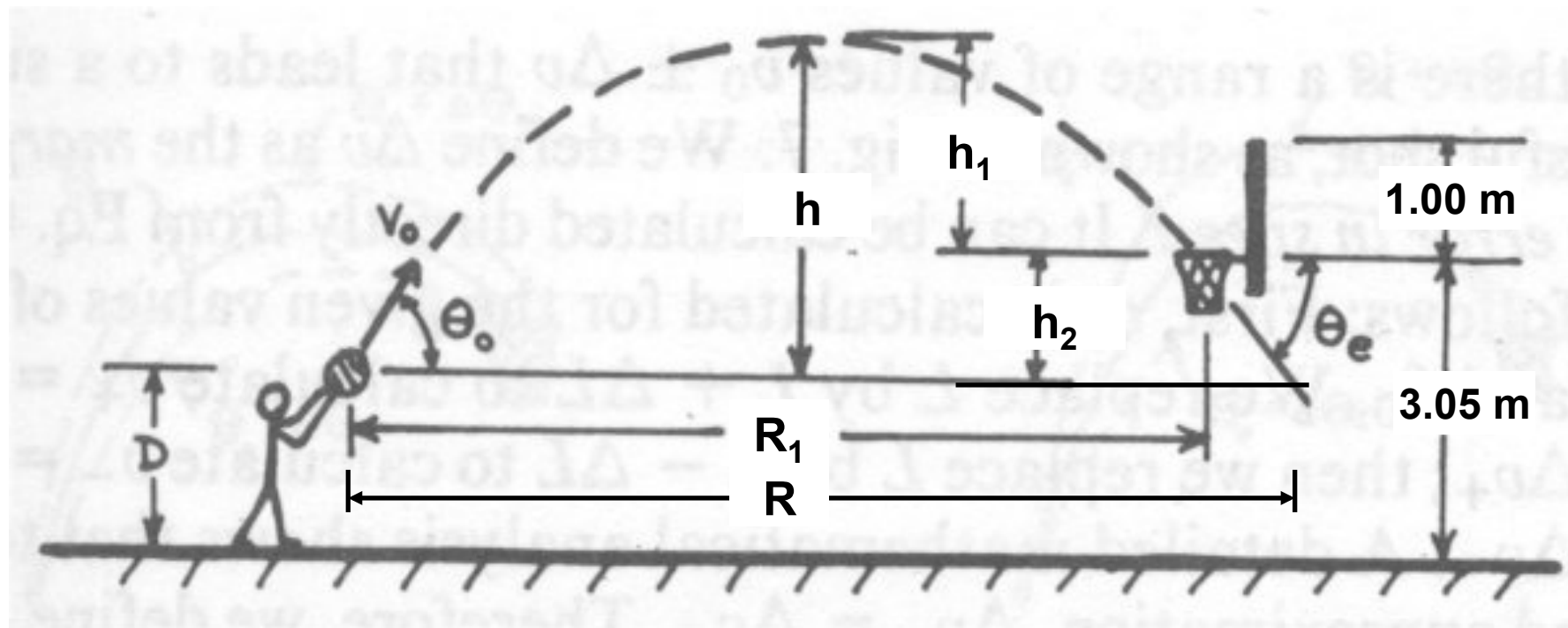
there are two solutions. If the launch angle is envisioned as an angle between 0° and 90° , then 2θ can take values between 0° and 180° . Both $\sin 2\theta_1$ and $\sin 2\theta_2$



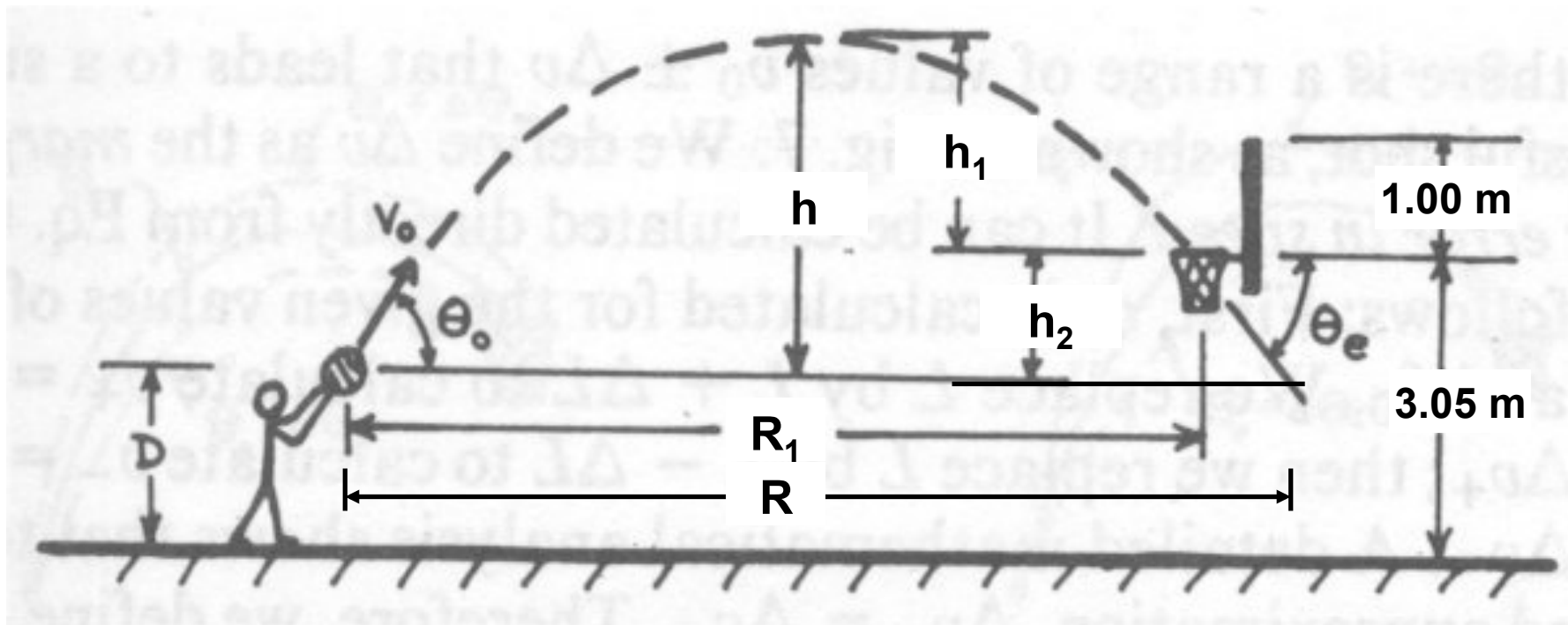
The two complementary launch angles approach each other at 45°



Apply to Basketball



Apply to Basketball



$$D = 1.75\text{ m}$$

$$h_2 = 3.05 - 1.75 = 1.3\text{ m}$$

$$\theta = 45^\circ$$

$$R_1 = 3.00\text{ m}$$

$$v_0 = 10\text{ m/s}$$