“Smart Basket Ball”

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References

- http://hyperphysics.phy-astr.gsu.edu/hbase/traj.html

The Motion Equations

These motion equations apply only in the case of constant acceleration.

$$y = \int v\,dt$$

$$= \int (v_0 + at)\,dt$$

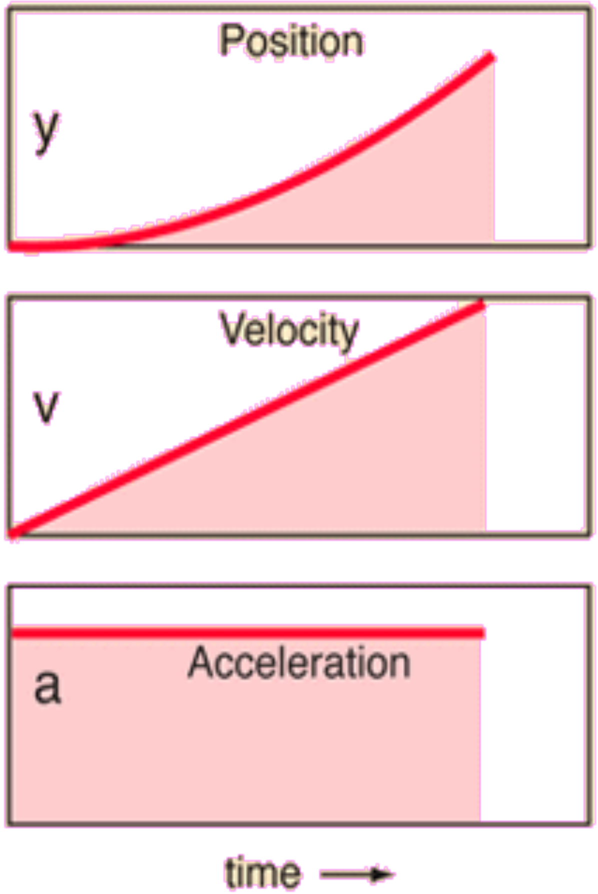
$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

Integrate velocity to get position

$$v = \int a\,dt = v_0 + at$$

Integrate acceleration to get velocity

$$a = \text{constant}$$



Motion relationships in one dimension.

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

Derivative of position is velocity

$$v = \frac{dy}{dt}$$

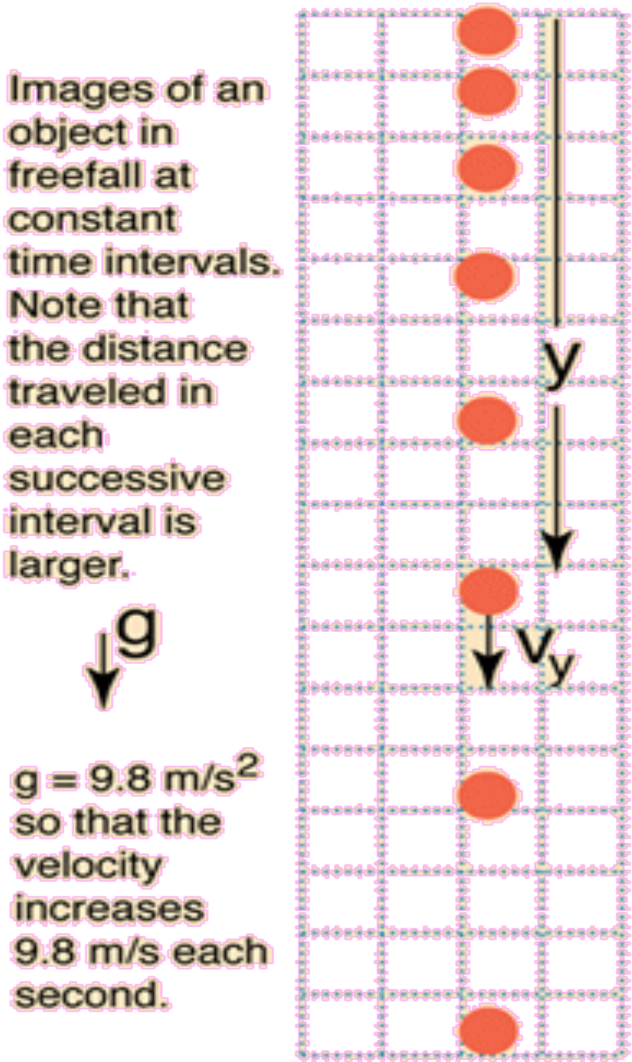
$$v = v_0 + at$$

Derivitive of velocity is acceleration

$$a = \frac{dv}{dt} = a$$

Free Fall

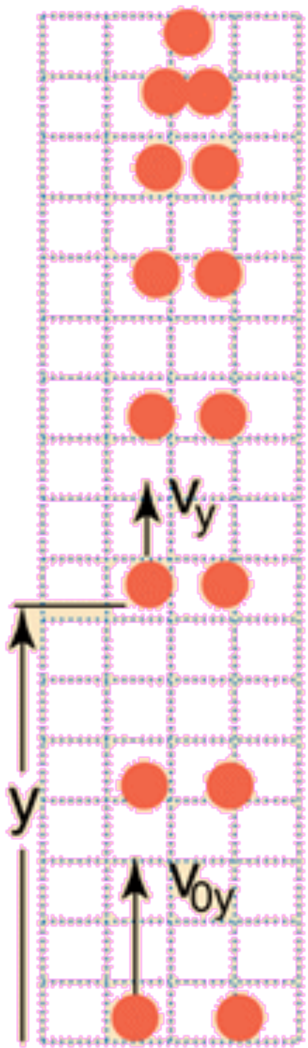
In the absence of frictional drag, an object near the surface of the earth will fall with the constant acceleration of gravity g.



$$v_y = gt$$
$$y = \frac{1}{2} g t^2$$

Taking $g = 9.8\ m/s^2$

Vertical Trajectory

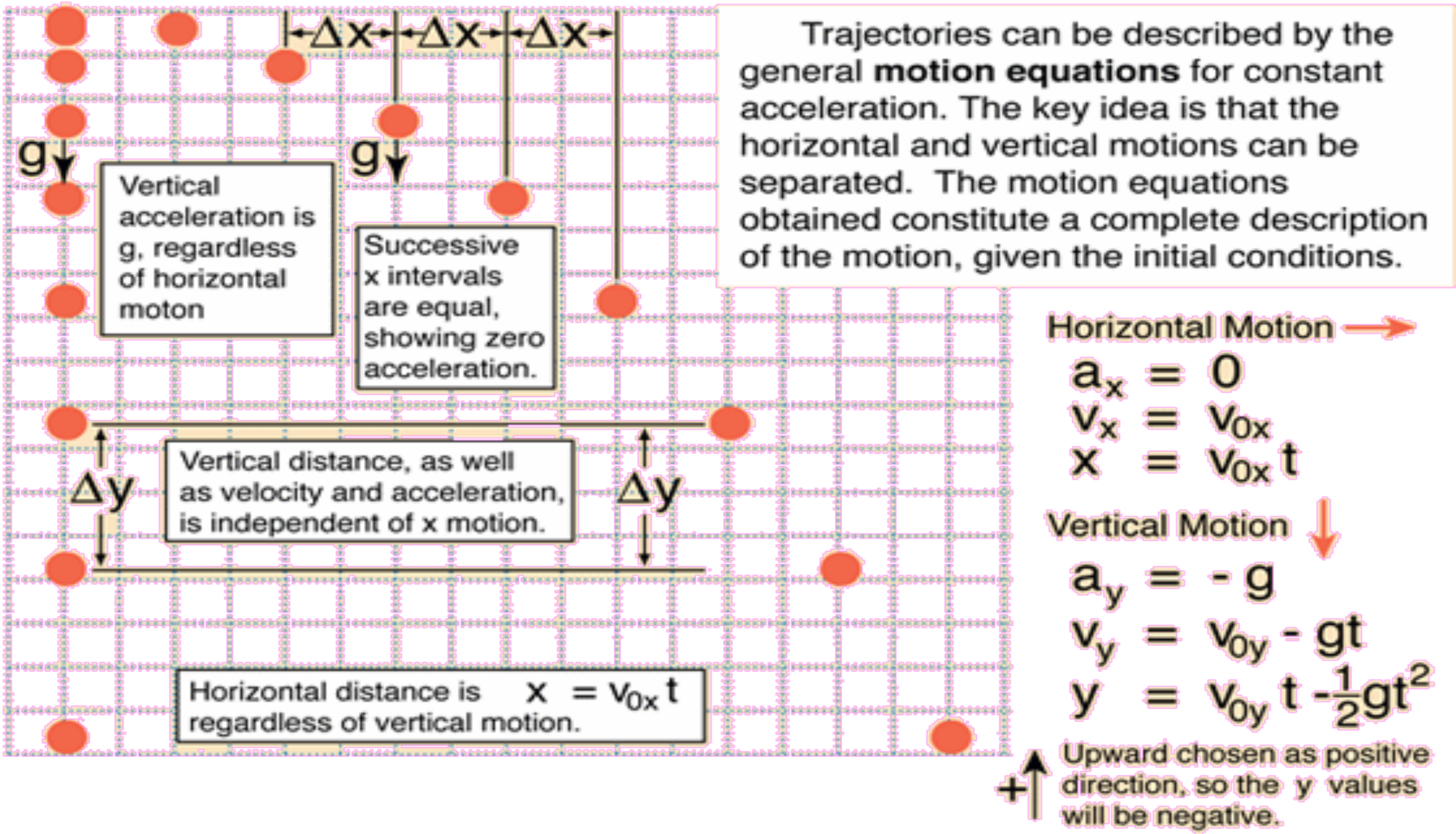


$$v_y = v_{0y} - gt$$

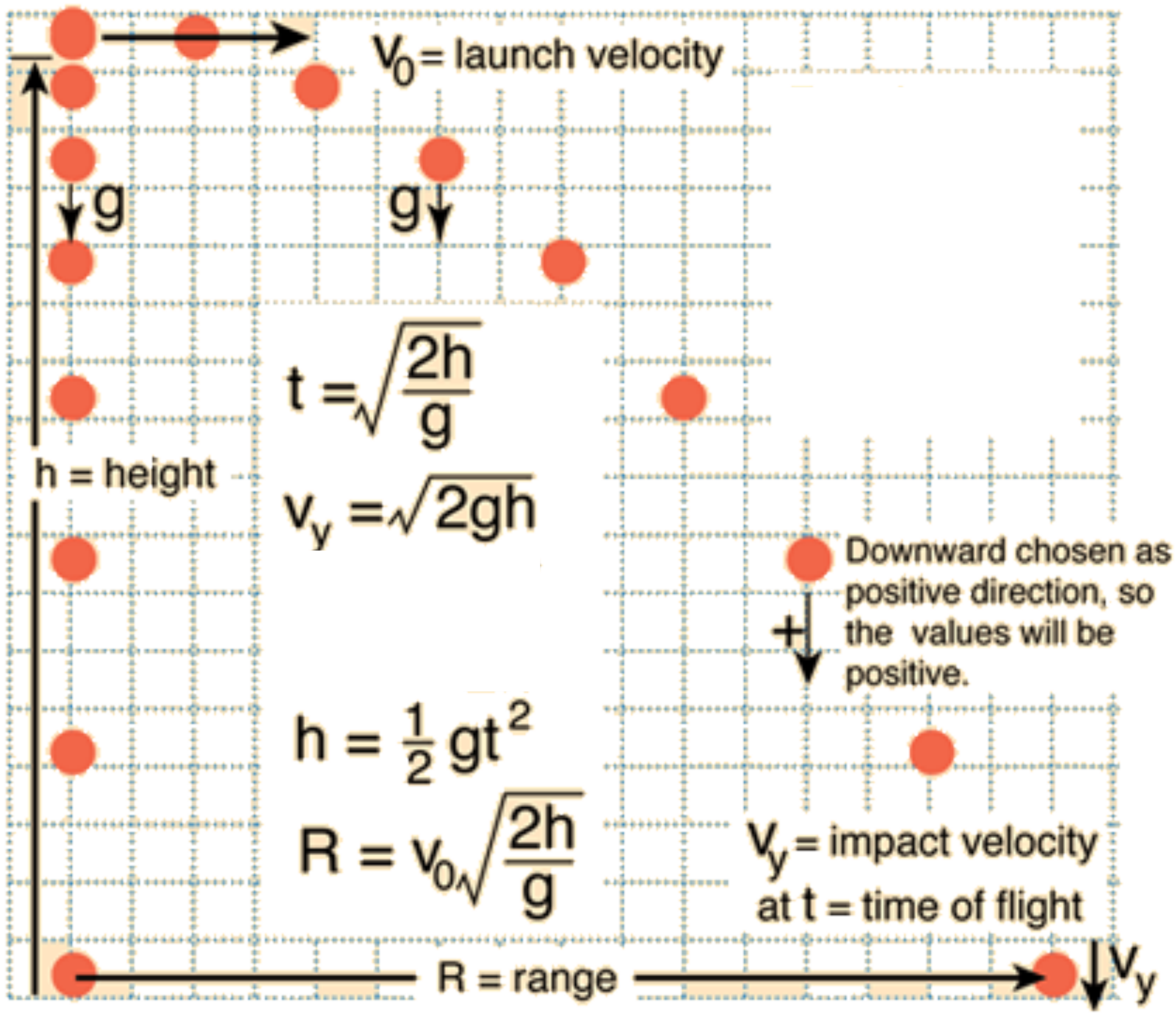
$$y = v_{0y}\, t - \frac{1}{2} g\, t^2$$

Taking $g = 9.8\ m/s^2$

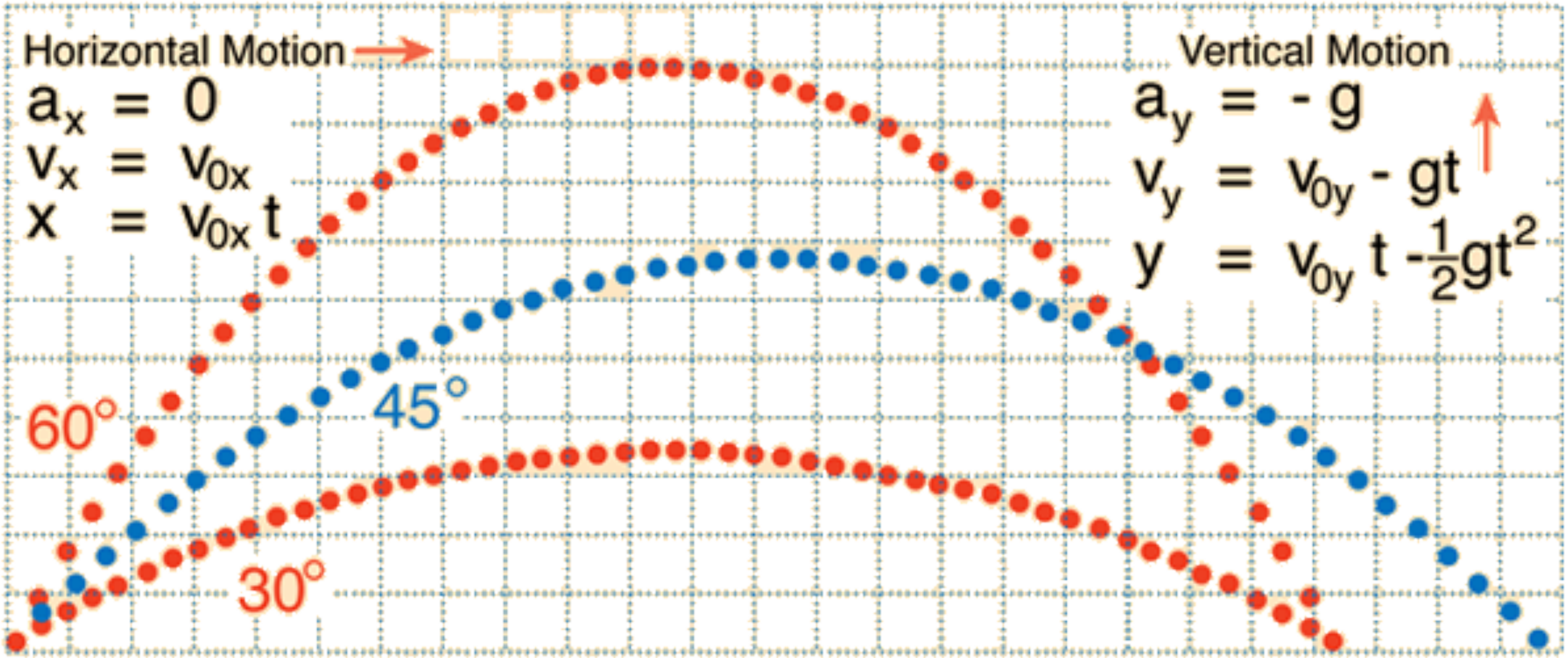
Horizontal Launch



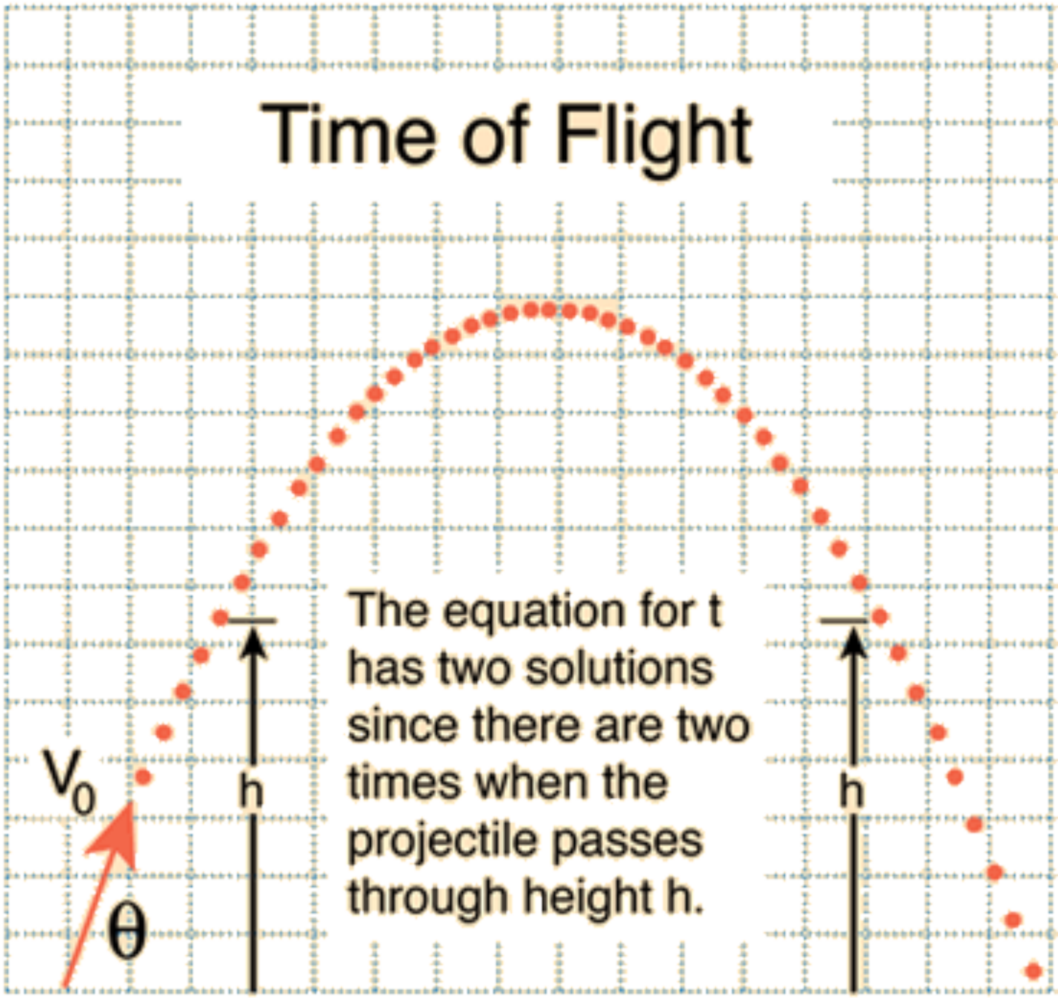
Horizontal Launch



General Ballistic Trajectory



Time of Flight



The basic motion equation

$$h = v_{0y}t - \frac{1}{2}gt^2$$

can be used to find the time of flight at height h, giving:*

$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2h}{g}}$$

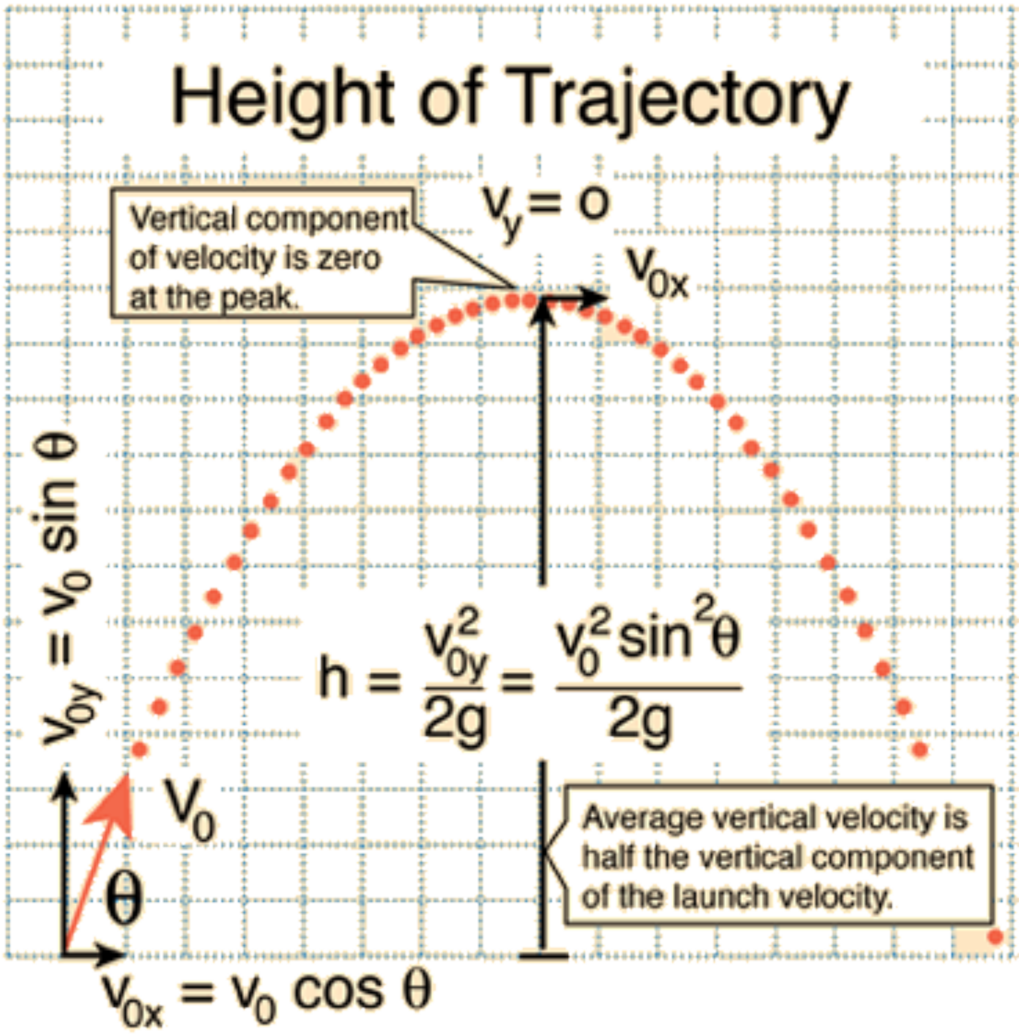
Note that there is no real solution if

$$\frac{2h}{g} > \frac{v_{0y}^2}{g^2} \quad \text{or } h > \frac{v_{0y}^2}{2g}$$

since such values of h are above the peak of the trajectory. For the value h=0:

$$t = 0 \text{ and } t = \frac{2v_{0y}}{g}$$

Height of Trajectory



The basic motion equation

$$y = \bar{v}_y t$$

can be used to find the height. The average vertical speed is:

$$\bar{v}_y = \frac{v_{0y} + 0}{2} = \frac{v_{0y}}{2}$$

The time at the peak is obtained by solving for the time at zero vertical speed:

$$0 = v_{0y} - g t_{peak}$$

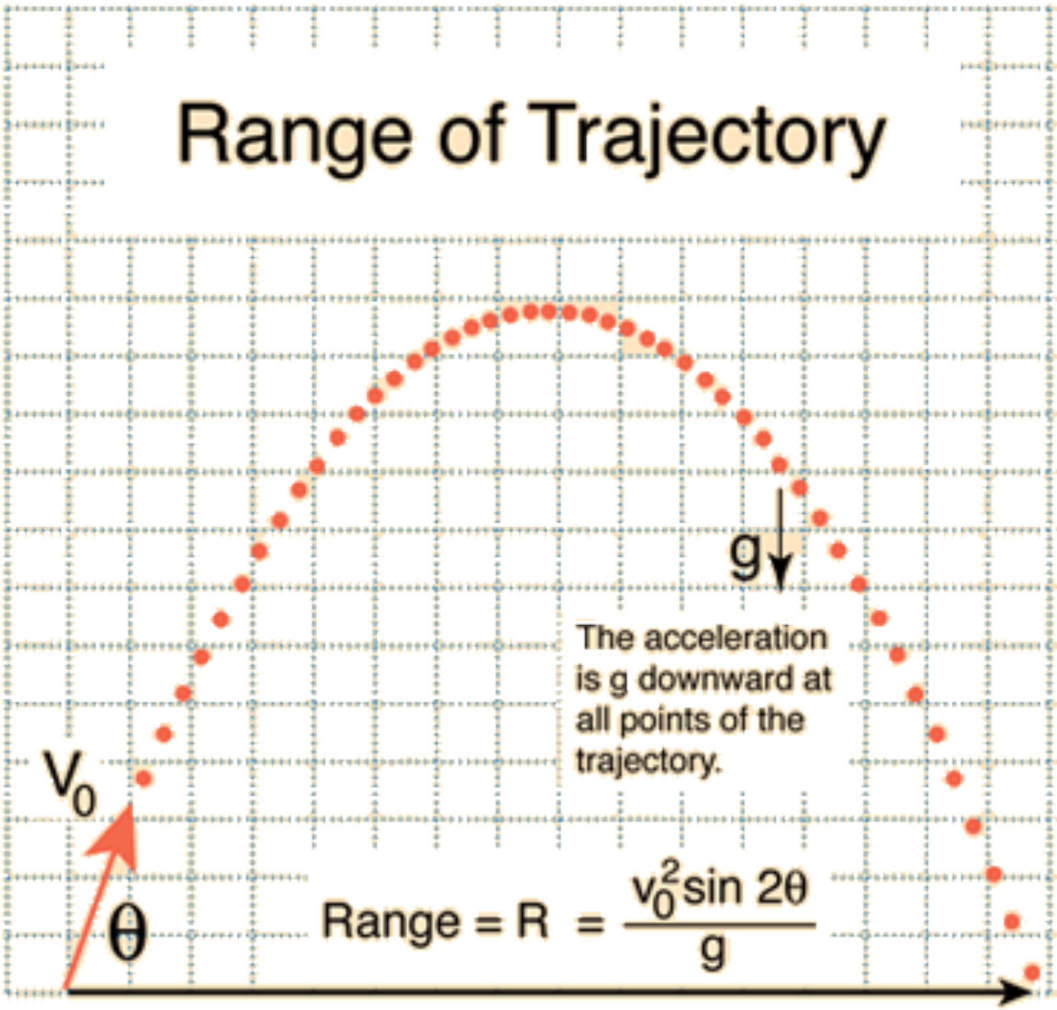
This gives:

$$t_{peak} = \frac{v_{0y}}{g}$$

and substituting:

$$h = y_{peak} = \frac{v_{0y}^2}{2g}$$

Range of Trajectory



The basic **motion equation**

$$x = v_{0x}\, t$$

can be used to find the range. By symmetry, the total **time of flight** is equal to twice the time at the peak:

$$t_{range} = 2t_{peak} = \frac{2v_{0y}}{g}$$

This gives:

$$R = \frac{2v_{0x}\, v_{0y}}{g}$$

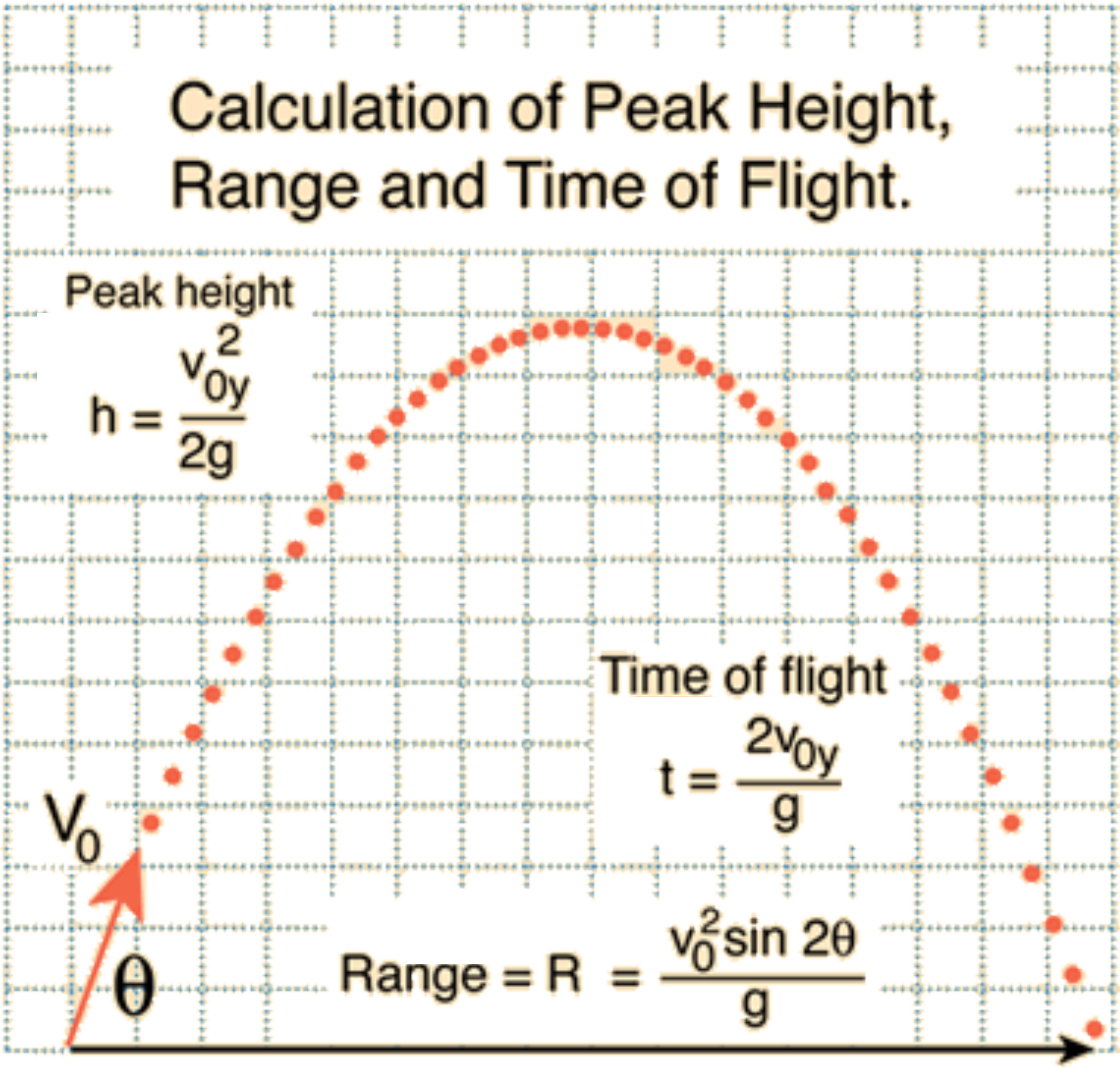
$$R = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

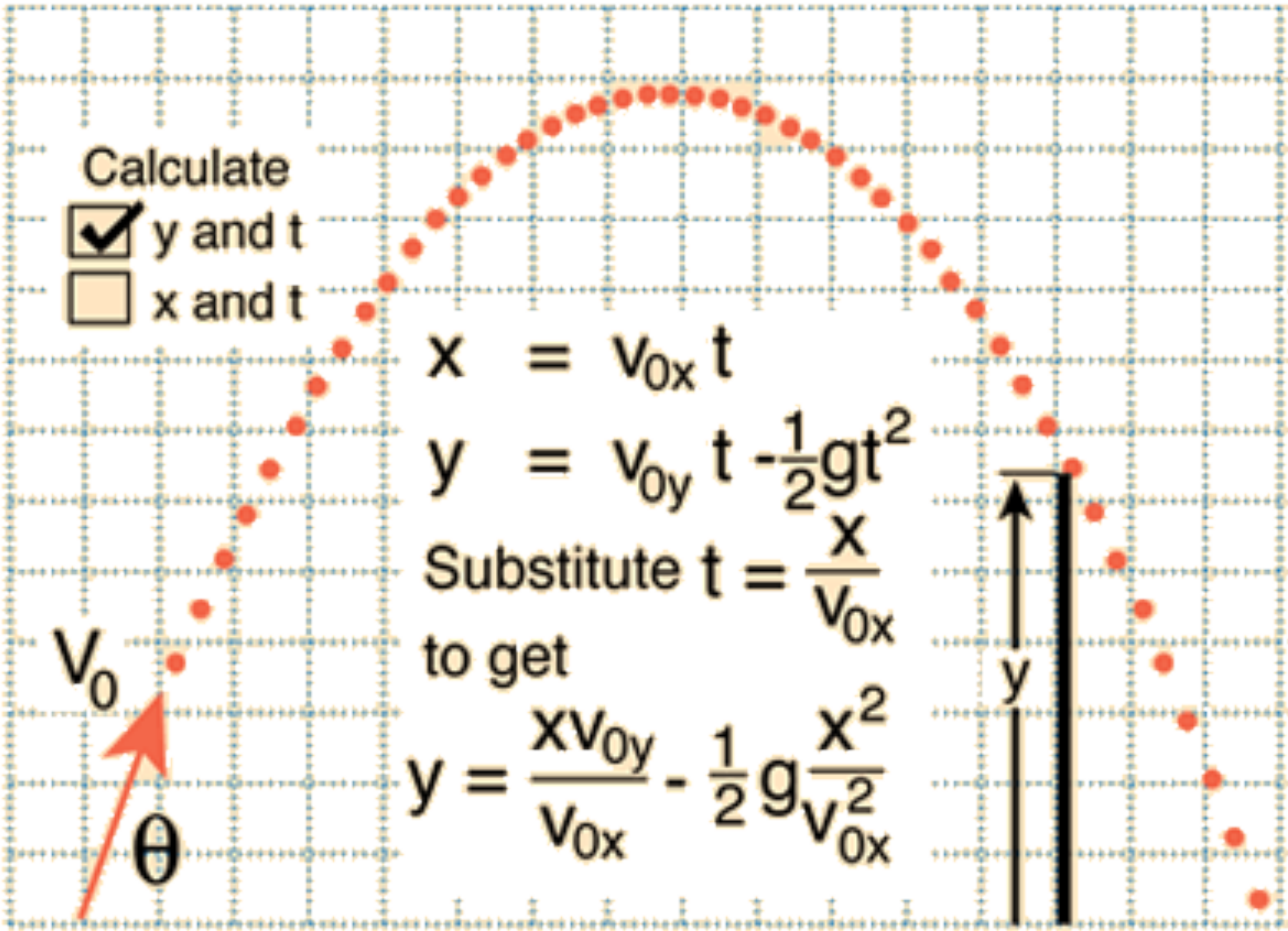
using the **trig identity**:

$$\sin 2\theta = 2\sin\theta \cos\theta.$$

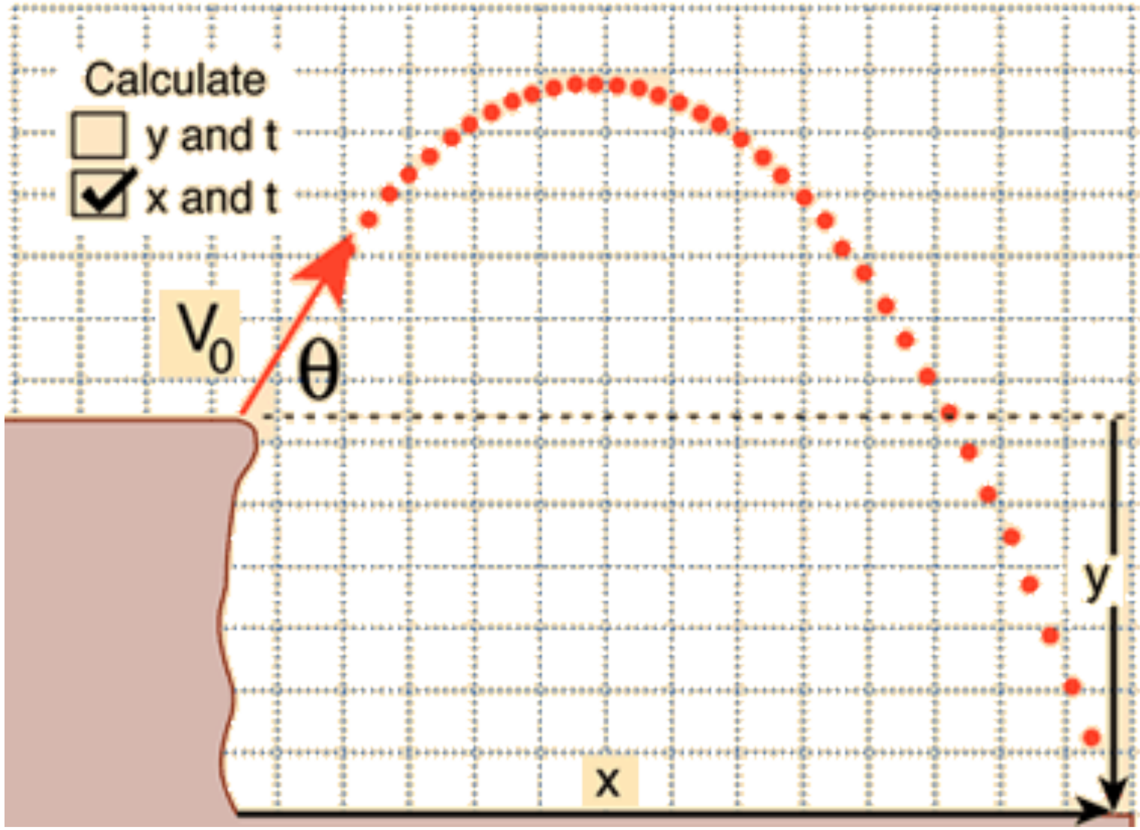
Peak Height, Range and Time of Flight



Express y in terms of x



Where will it land?



$$x = v_{0x}\, t$$

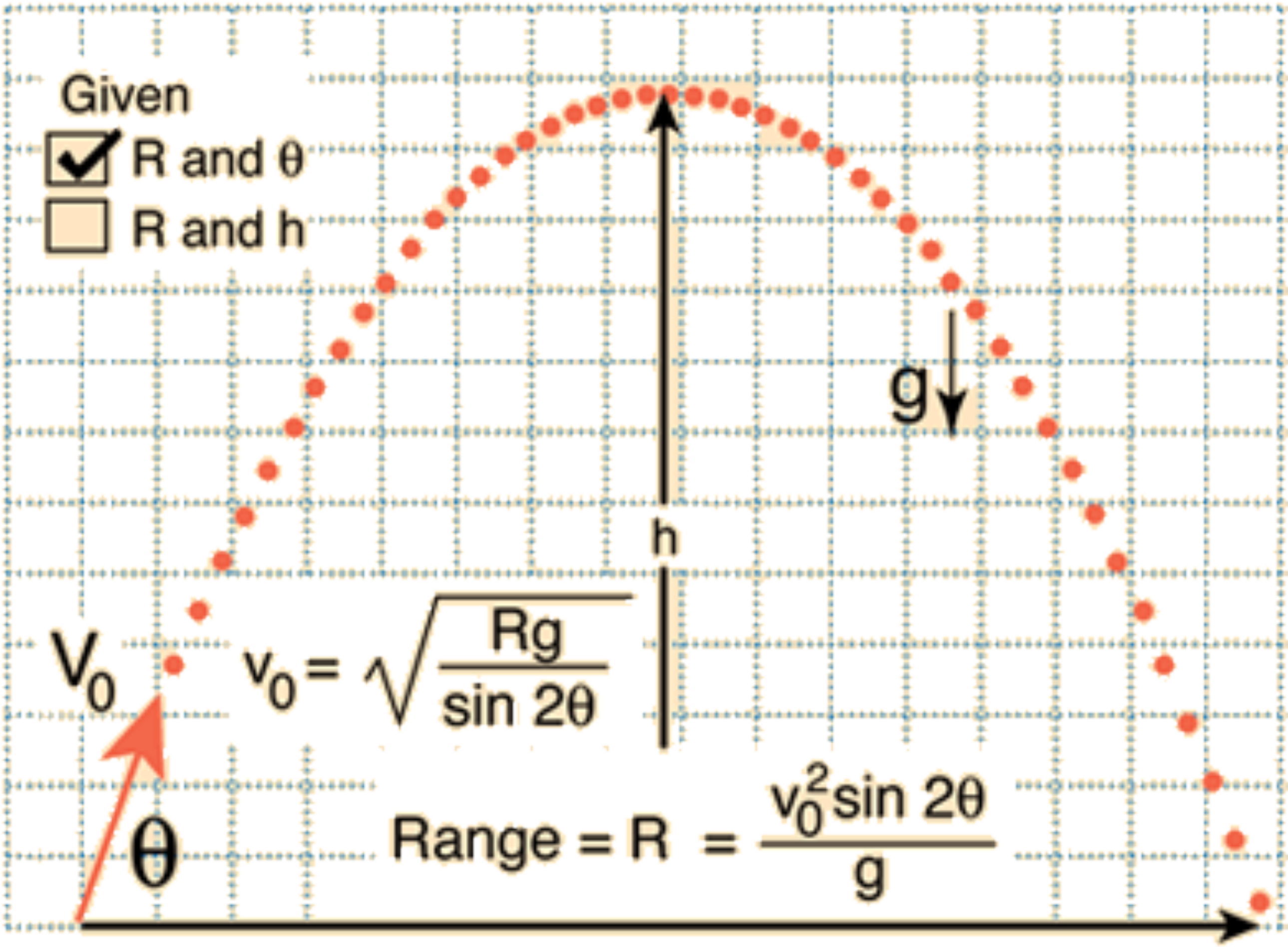
$$y = v_{0y}\, t - \frac{1}{2} g t^2$$

Using the **quadratic formula** to solve for t gives two values of time for a given value of y:

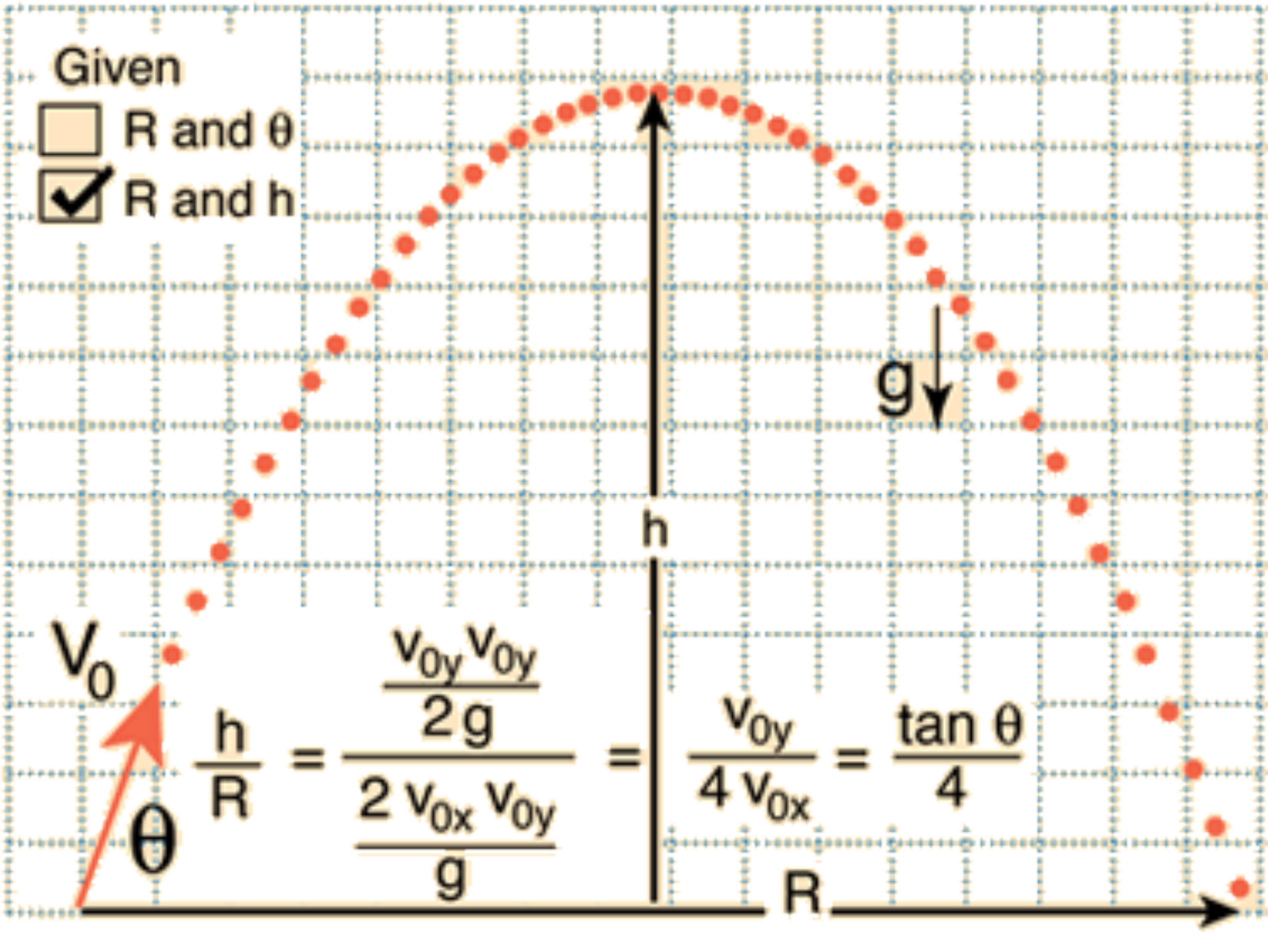
$$t = \frac{v_{0y}}{g} \pm \sqrt{\frac{v_{0y}^2}{g^2} - \frac{2y}{g}}$$

Substitution of the two time values gives the two values of x corresponding to a given height y.

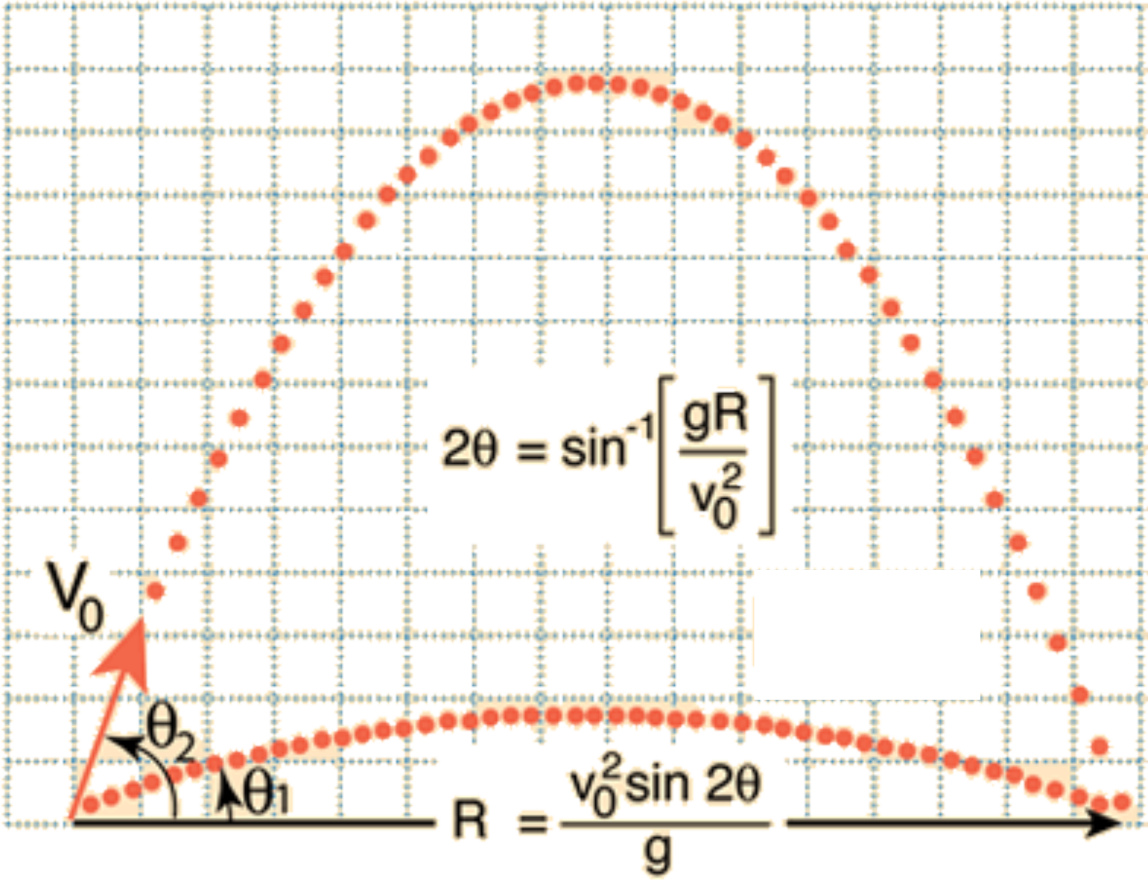
Launch velocity I



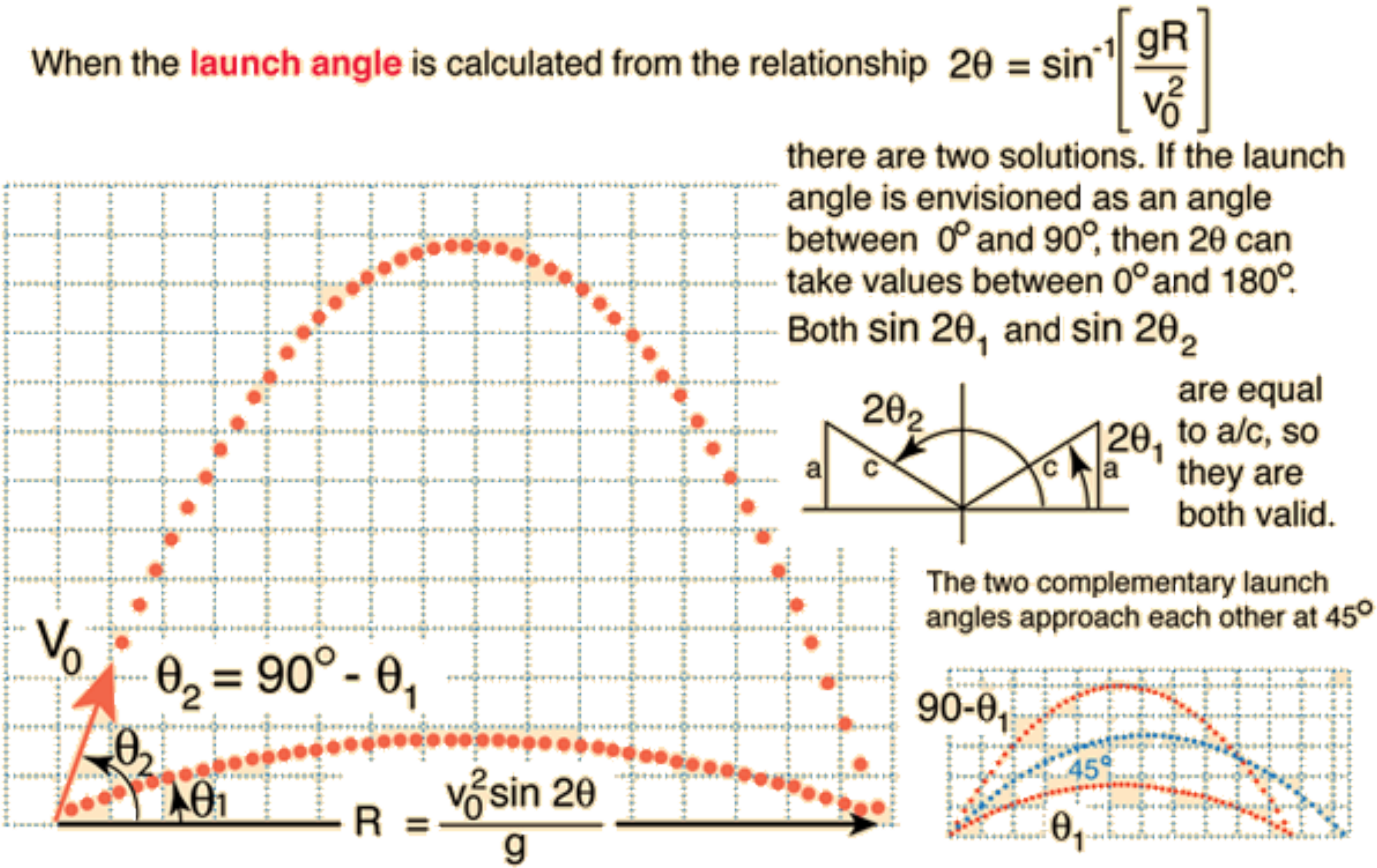
Launch Velocity II



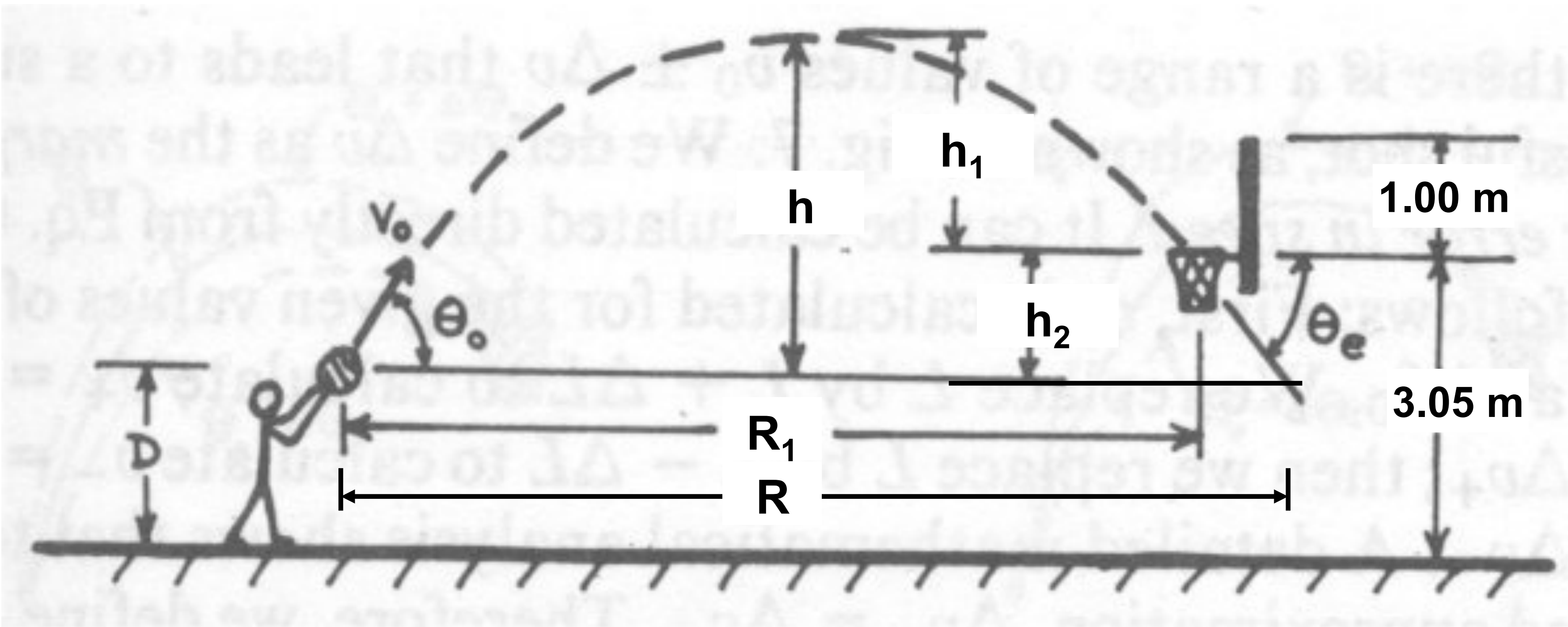
Angle of Launch I



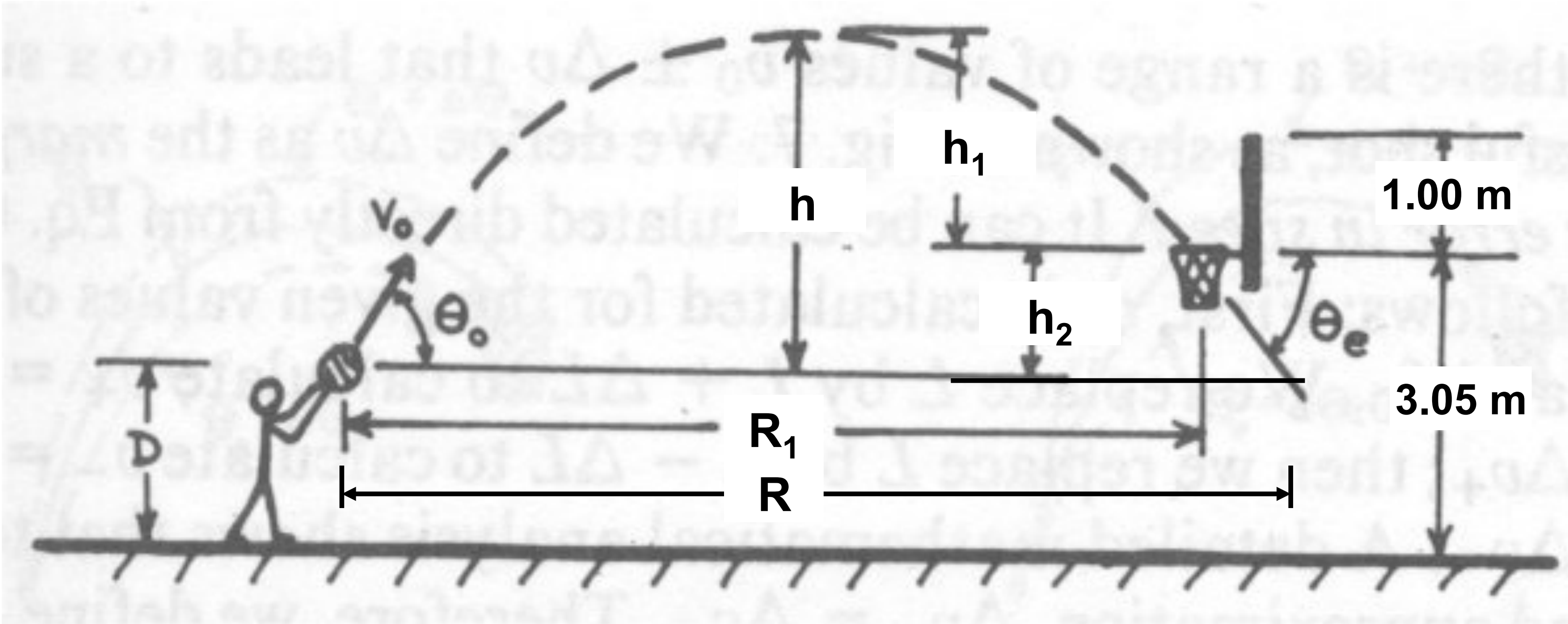
Angle of Launch II



Apply to Basket Ball



Apply to Basket Ball



$D = 1.75$ m

$\Theta = 45^o$

$Vo = 10$ m/s

$h_2 = 3.05 - 1.75 = 1.3$ m

R1= 3.00m