1 Theorem 1 partial proof : Proofs of each basic cases

s' enriches s, $K(W_i), s \models p \Leftrightarrow K(W_{i+1}), s' \models p$, p is an atomic proposition that is not concerned with the increment.

Proof. ($\Rightarrow$) By definition, if s' enriches s, s' contains a greater set of atomic propositions than s. As $s \models p$, p being an atomic proposition of s, then p is an atomic proposition of s', hence $s' \models p$.

($\Leftarrow$) If p is not a property concerned with the increment and $s' \in S_{K(W_{i+1})}$ enriches $s \in S_{K(W_i)}$, then $K(W_{i+1}), s' \models p \Rightarrow K(W_i) \models p$.

s' enriches s, $K(W_i), s \models EXp \Leftrightarrow K(W_{i+1}), s' \models (e\_qt \wedge EXp)$ and p in $AP_{K(W_i)}$.

Proof. ($\Rightarrow$) If $s \models EXp$, there exists a state $t = (u, c) \in S_{K(W_i)}$ such that $s \to t$ and $t \models p$. Let be a state s' enriching s with e_qt; by Corollary 1 item 2 there exists $t' = (u', c') \in S_{K(W_{i+1})}$ such that $s' \to t'$, $u = u'$, and $proj(c', I_i) = c$. Hence $K(W_{i+1}), t' \models p$ and $K(W_{i+1}), s' \models e\_qt \wedge EXp$.

($\Leftarrow$) Let be $K(W_{i+1})$,$s' \models e\_qt \wedge EXp$ and s' enriches s with e_qt; let be $t' = (u', c')$ such that $s' \to t'$ and $t' \models p$. By Corollary 1 item 4, there exists $t = (u, c) \in S_{K(W_i)}$, such that $u = u'$, and $proj(c', I_i) = c$, then as $p \in AP_{K(W_i)}$, $t \models p$. Moreover, s' simulates s, hence $K(W_i), s \models EXp$.

s' enriches s, $K(W_i), s \models EFp \Leftrightarrow K(W_{i+1}), s' \models E[e\_qtUp]$.

Proof. ($\Rightarrow$) If $s \models p$, $s' \models p$ then $s' \models EFp$.
If $s \not\models p$, there exists a path σ in $K(W_i) : \sigma = s \to t \to \ldots \to r$, such that $r \models p$. By Corollary 1 item 1, there exists a path σ' in $K(W_{i+1}) : s' \to t' \to \ldots \to r'$ such that s' enriches s with e_qt, t' enriches t with e_qt,..., and r' enriches r then $r \models p$, hence $s' \models E[e\_qtUp]$.

($\Leftarrow$) Let $K(W_{i+1}), s' \models E[e\_qtUp]$ and s' enriches s with e_qt. There exists a path σ' in $K(W_{i+1}) : s' \to t' \to \ldots \to r'$ such that for all $s' \leq u' < r'$, $u' \models e\_qt$ and $r' \models p$. By Corollary 1 item 4, there exists a path σ in $K(W_i) : s \to t \to \ldots \to r$ such that for all $s \leq u < r$ u' simulates u and r' enriches r. Hence $r \models p$ if p is not concerned by the increment and $K(W_i), s \models EFp$.

s' enriches s, $K(W_i), s \models EGp \Leftrightarrow K(W_{i+1}), s' \models EG(e\_qt \wedge p)$.

Proof. ($\Rightarrow$) If $K(W_i), s \models EGp$ there exists an infinite path σ in $K(W_i)$ s.t. $s \to t \to \ldots \to r \to \ldots$, and such that $s \models p$, $t \models p$, ..., $r \models p$, ... By Corollary 1 item 1, there exists an infinite path σ' in $K(W_{i+1}) : s' \to t' \to \ldots \to r' \to \ldots$, such that $s' \models p \wedge e\_qt$, $t' \models p \wedge e\_qt$, ..., $r' \models p \wedge e\_qt$,... : $s' \models EG(e\_qt \wedge p)$.

($\Leftarrow$) Let $K(W_{i+1}), s' \models EG(e\_qt \wedge p)$ and s' enriches s with e_qt, $s \in S_{K(W_i)}$. There exists an infinite path σ' in $K(W_{i+1})$ $s' \to t' \to \ldots \to r' \to \ldots$ such that for all state u' of this path $u' \models p \wedge e\_qt$. By Corollary 1 item 4, there exists an infinite path σ in $K(W_i) : s \to t \to \ldots \to r \to \ldots$ such that s' enriches s with e_qt, t' enriches t with e_qt ..., hence if p is not concerned by the increment, $s \models p$, $t \models p$. Hence, $K(W_i), s \models EGp$.

s' enriches s, $K(W_i), s \models E[pUq] \Leftrightarrow K(W_{i+1}), s' \models E[e\_qt \wedge pUq]$.

Proof. ($\Rightarrow$) If $s \models q$, then $s' \models q$ hence $s' \models E[(e\_qt \wedge p)Uq]$. If $s \not\models q$, there exists an infinite path $\sigma : s \to t \to \ldots \to r \to \ldots$, such that $s \models p$, $t \models p$, ..., $r \models q$. Let s' enriches s with e_qt, by Corollary 1 item 1, there exists an infinite path σ' in $K(W_{i+1}) : s' \to t' \to \ldots \to r' \to \ldots$, such that $s' \models p \wedge e\_qt$, $t' \models p \wedge e\_qt$, ..., r' enriches r and $r' \models q$. Hence, $s' \models E[(e\_qt \wedge p)Uq]$.

($\Leftarrow$) Let $K(W_{i+1}), s' \models E[(e\_qt \wedge p)Uq]$ and s' enriches s with e_qt, $s \in S_{K(W_i)}$. There exists an infinite path σ' in $K(W_{i+1}) : s' \to t' \to \ldots \to r' \ldots$ such that $s' \models p \wedge e\_qt$, $t' \models p \wedge e\_qt$, ..., $r' \models q$. By Corollary 1 item: 4, there exists an infinite path in $K(W_i)$ $\sigma = s \to t \to \ldots \to r \to \ldots$ such that s' enriches s with e_qt, t' enriches t with e_qt, ..., r' enriches r. If p and q are not concerned by the increment, σ satisfies $[pUq]$, hence $K(W_i), s \models E[pUq]$.

s' enriches s, $K(W_i), s \models AXp \Leftrightarrow K(W_{i+1}), s' \models (e\_qt \Rightarrow AXp)$.

Proof. ($\Rightarrow$) In $K(W_i)$, p holds for all successors r of s. If s' enriches s with e_qt, for all successors r' of s', there exists a successor r of s such that r' enriches r (Corollary 1 item 3) hence $r' \models p$. Else (if s' enriches s with e_act), $s' \models e\_act$, and $e\_act = \neg e\_qt$. Hence $s' \models e\_qt \Rightarrow AXp$.

($\Leftarrow$) $K(W_{i+1}), s' \models e\_qt \Rightarrow AXp$. s' enriches s, either with e_act (nothing to be said), or with e_qt (hypothesis). By Corollary 1 item 4, all successors of s' enriches states that are successors of s in $K(W_i)$ (with e_qt or e_act). All of them must verify p, hence $s \models AXp$.

s' enriches s, $K(W_i), s \models AFp \Leftrightarrow K(W_{i+1}), s' \models AF(e\_act \vee p)$.

Proof. ($\Rightarrow$) In $K(W_i)$ for all infinite path $\sigma = s_0, \ldots s_n \ldots$, there exists a state s_k, $0 \leq k \leq n$ in which p is true. From Corollary 1 item 1, there exists some path in $K(W_{i+1})$ $\sigma' = s'_0, \ldots, s'_i, \ldots s'_n$, such

that all the states s'_i enriches s_i with e_qt. Moreover, by constructing $K(W_{i+1})$ we have that there doesn't exist any transition t'_k from a state s'_k in σ' to a state s'_{k+1} labeled with e_qt and which is not in the following induction hypothesis :
If $s'_k \in \sigma'$ then $s'_k \models AF(p \vee e\_act)$
From s'_0, there exists a path such that all the states verify e_qt and AFp, hence $s'_0 \models AF(p \vee e\_act)$.
Let $s'_k \in \sigma'$, the set of these successors are as :
$s'_{k+1} = s_{k+1} \wedge e\_act$ and thus verify AF(p ∨ e_act) or $s'_{k+1} = s_{k+1} \wedge e\_qt$,the transition from $s_k \rightarrow s_{k+1}$ is in σ' and thus verify $AF(p \vee e\_act)$ (induction hypothesis).

(⇐) Let s'_0 be a state in $S_{K(W_{i+1})}$ s.t. $s'_0 \models AF(e\_act \vee p)$, and s'_0 enriches s_0 with e_qt.

1. if $s'_0 \models p$ then $s_0 \models p$, hence $s_0 \models AF(p)$.
2. if $s'_0 \not\models p$ then there exists three categories of successors : $t' \models e\_qt \wedge p$; $t'' \models e\_act$; $r' \models e\_qt \wedge \neg p$ and $r' \models AF(e\_act \vee p)$. By Corollary 1 item 4, there exists t and r in $S_{K(W_i)}$ s.t. they are successors of s_0 and $t \models p$ and one can prove by induction that $r \models AFp$. Hence $s_0 \models AFp$.

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s' enriches s, $K(W_i), s \models A[pUq] \Leftrightarrow K(W_{i+1}), s' \models A[pU((e\_act \wedge p) \vee q)]$.

Proof. (⇒) 1. Let be $s_0 \in S_{K(W_i)}$, if $s_0 \models q$, then we have $s_0 \models A[pUq]$. Let be $s'_0 \in S_{K(W_{i+1})}$, s'_0 enriches s_0 with e_qt then $s'_0 \models q$, hence $s'_0 \models A[pU((e\_act \wedge p) \vee q)]$.

2. If $s_0 \not\models q$, $s_0 \models p$ and all its successors $Succ(s_0)$ verify $A[pUq]$. Let r in $Succ(s_0)$ be s.t. $r \models q$, then we have r' enriches r with e_qt verifies q. Hence $r' \models A[pU((e\_act \wedge p) \vee q)]$. Let t in $Succ(s_0)$ be s.t. $t \models p$, then $\exists t', t''$ in $S_{K(W_{i+1})}$ s.t $t' \models p \wedge e\_qt$ and $t'' \models p \wedge e\_act$. By induction, one can prove $t' \models A[pU((e\_act \wedge p) \vee q)]$. Hence all successors of s'_0 verify $s' \models, A[pU((e\_act \wedge p) \vee q)]$ and s'_0 verifies it also.

(⇐) Let s'_0 be a state in $S_{K(W_{i+1})}$ such that $s'_0 \models A[pU((e\_act \wedge p) \vee q)]$ and s'_0 enriches s_0 with e_qt.

1. If $s'_0 \models q$ then $s_0 \models q$ and $s_0 \models A[pUq]$.
2. If $s'_0 \models p \wedge e\_act$, contradiction with the hypothesis.
3. If $s'_0 \models p$ then there exists 3 categories of successor states: (see Figure **??**) $t' \models p \wedge e\_qt$, $t'' \models p \wedge e\_act$ and $r' \models q$. Moreover, t' and t'' verify $A[pU((e\_act \wedge p) \vee q)]$. By Corollary 1 item 4, there exists t and r in $S_{K(W_i)}$ s.t. they are successors of s_0 and $t \models p$ and $r \models q$. By incremental construction s_0 can not have a successor that verify $(\neg p \wedge \neg q)$ and one can prove, by induction that t verifies $A[pUq]$. Hence s_0 verifies $A[pUq]$.

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s' enriches s, $K(W_i), s \models AGp \Leftrightarrow K(W_{i+1}), s' \models A[pW(e\_act \wedge p)]$.

Proof. (⇒) 1. *(In $K(W_{i+1})$, a state that do not verify p belongs to an added behaviour).* Let be $t' \in S_{K(W_{i+1})}$, $t' \models \overline{p} \wedge e\_qt$ and $\sigma' = s' \ldots t'$. $t' = (u', c')$ doesn't simulate a state in $S_{K(W_i)}$ ($\forall s \in S_i$ $s \models p$). u' corresponds to an added state into $W_{i+1} (\in \Sigma_+)$. Hence, t' is only reachable from a sequence having a state where e_act holds (Corollary 1 item 3).

2. *In $K(W_{i+1})$, along paths reached from the initial state, all states verify e_qt and p until a state where e_act holds is reached.* Let be s' such that s' enriches s and a sequence $\sigma' = s' \ldots r' \ldots t'$ with $s' < r' \leq t'$, if $\nexists m'$ labeled with e_act such that $m' < r'$ then $r' \models p \wedge e\_qt$ or $r' \models p \wedge e\_act$.
3. *Infinite paths in $K(W_i)$ correspond to infinite paths labeled with e_qt in $K(W_{i+1})$.* By Corollary 1 item 1, if there exists some infinite path in $K(W_i)$, there exists some infinite path in $K(W_{i+1})$ labeled with e_qt. Then there exists some infinite path in $K(W_{i+1})$ which verify $p \wedge e\_qt$.
We have thus $s' \models A[pW(e\_act \wedge p)]$

(⇐) Let s'_0 be a state in $S_{K(W_{i+1})}$ such that $s'_0 \models A[pU((e\_act \wedge p) \vee q)]$ and s'_0 enriches s_0 with e_qt. All successor of s'_0 are such that : $t' \models p \wedge e\_qt$ or $t'' \models p \wedge e\_act$ and verifies $A[pU((e\_act \wedge p) \vee q)]$. If p is not concerned by the increment, By Corollary 1 item 4 there exists $t \in S_{K(W_i)}$ such that $t \models p$. By incremental construction s'_0 can not have a successor were $\neg p$ holds and one can prove, by induction that t verifies AGp. Hence s_0 verifies AGp.

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s' enriches s, $K(W_i), s \models A[pWq] \Leftrightarrow K(W_{i+1}), s' \models A[pW((e\_act \wedge p) \vee q)]$.

Proof. (⇒) $s \models A[pWq]$ then all paths from s are such they verified pUq or Gp. In the first case, we use the same reasoning as $A[pUq]$. In the second case, by Corollary 1 item 1 these paths exist in $K(W_{i+1})$. The divergent behaviours are labeling with e_act (Corollary 1 item 3). We have thus $K(W_{i+1}), s' \models A[pW((e\_act \wedge p) \vee q)]$

(⇐) Same reasoning as $A[pU((e\_act \wedge p) \vee q)]$.

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s' enriches s, $K(W_i), s \models \neg p \Leftrightarrow K(W_{i+1}), s' \models \neg p$.

Proof. The proof proceeds as the one concerning positive atomic propositions.

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s' enriches s, $K(W_i), s \models \neg\Psi \Leftrightarrow K(W_{i+1}), s' \models \neg\Psi'$.

Proof. The proof is performed by first transforming the formulae into positive form and then applying the transformation proven above. We compare the result with the one obtain by transforming directly the negative form. The weak until operator (W) is related to the strong until operator (U) by the following equivalences:

$$A[pWq] = \neg E[\neg qU(\neg p \wedge \neg q)]$$

$$E[pUq] = \neg A[\neg qW(\neg p \wedge \neg q)]$$

We recall here that $\neg e\_act = e\_qt$

$$\begin{aligned}
(\neg EXp)' &= (AX\neg p)' \\
&= e_qt \Rightarrow AX\neg p \\
\neg(EXp)' &= \neg(e_qt \wedge EX\neg p) \\
&= \neg e_qt \vee AX\neg p \\
&= e_qt \Rightarrow AX\neg p \\
Hence \quad (\neg EXp)' &= \neg(EXp)' \qquad (1)
\end{aligned}$$

$$\begin{aligned}
(\neg EFp)' &= (AG\neg p)' \\
&= A[\neg pW(e_act \wedge \neg p)] \\
\neg(EFp)' &= \neg E[e_qtUp] \\
&= A[\neg pW(e_act \wedge \neg p)] \\
Hence \quad (\neg EFp)' &= \neg(EFp)' \qquad (2)
\end{aligned}$$

$$\begin{aligned}
(\neg EGp)' &= (AF\neg p)' \\
&= AF(e_act \vee \neg p) \\
\neg(EGp)' &= \neg EG(e_qt \wedge p) \\
&= AF(e_act \vee \neg p) \\
Hence \quad (\neg EGp)' &= \neg(EGp)' \qquad (3)
\end{aligned}$$

$$\begin{aligned}
(\neg E[pUq])' &= A[\neg qW(\neq p \wedge \neg p)]' \\
&= A[\neg qW((e_act \wedge \neg q) \vee (\neg p \wedge \neg q)] \\
\neg(E[pUq])' &= \neg E[(e_qt \wedge p)Uq] \\
&= A[\neg qW\neg(\neg(e_qt \wedge p) \wedge \neg q)] \\
&= A[\neg qW\neg((e_act \vee \neg p) \wedge \neg q)] \\
&= A[\neg qW((e_act \wedge \neg q) \vee (\neg p \wedge \neg q)] \\
Hence \quad (\neg E[pUq])' &= \neg(E[pUq])' \qquad (4)
\end{aligned}$$

$$\begin{aligned}
(\neg E[pWq])' &= A[\neg qU(\neq p \wedge \neg p)]' \\
&= A[\neg qU((e_act \wedge \neg q) \vee (\neg p \wedge \neg q)] \\
\neg(E[pWq])' &= \neg E[(e_qt \wedge p)Wq] \\
&= A[\neg qU\neg(\neg(e_qt \wedge p) \wedge \neg q)] \\
&= A[\neg qU\neg((e_act \vee \neg p) \wedge \neg q)] \\
&= A[\neg qU((e_act \wedge \neg q) \vee (\neg p \wedge \neg q)] \\
Hence \quad (\neg E[pWq])' &= \neg(E[pWq])' \qquad (5)
\end{aligned}$$

$$\begin{aligned}
(\neg AXp)' &= (EX\neg p)' \\
&= e_qt \wedge EX\neg p \\
\neg(AXp)' &= \neg(e_qt \Rightarrow AX\neg p) \\
&= e_qt \wedge EX\neg p \\
Hence \quad (\neg AXp)' &= \neg(AXp)' \qquad (6)
\end{aligned}$$

$$\begin{aligned}
(\neg AFp)' &= (EG\neg p)' \\
&= EG(e_qt \wedge \neg p) \\
\neg(AFp)' &= \neg AF(e_act \vee p) \\
&= EG(e_qt \wedge \neg p) \\
Hence \quad (\neg AFp)' &= \neg(AFp)' \qquad (7)
\end{aligned}$$

$$\begin{aligned}
(\neg AGp)' &= (EF\neg p)' \\
&= E[e_qtU\neg p] \\
\neg(AGp)' &= \neg A[pW(e_act \wedge p)] \\
&= E[\neg(e_act \wedge p)U(\neg p \wedge \neg(e_act \wedge p))] \\
&= E[(e_qt \vee \neg p)U\neg p] \\
&= E[e_qtU\neg p] \\
Hence \quad (\neg AGp)' &= \neg(AGp)' \qquad (8)
\end{aligned}$$

$$\begin{aligned}
(\neg A[pUq])' &= (E[\neg qW(\neg p \wedge \neg q)])' \\
&= E[(e_qt \wedge \neg q)W(\neg p \wedge \neg q)] \\
\neg(A[pUq])' &= \neg A[pU((e_act \wedge p) \vee q)] \\
&= E[\neg((e_act \wedge p) \vee q)W(\neg p \wedge \neg((e_act \wedge p) \vee q))] \\
&= E[((_act \wedge \neg q) \vee (\neg p \wedge \neg q))W(\neg p \wedge \neg q)] \\
&= E[(e_qt \wedge \neg q)W(\neg p \wedge \neg q)] \\
Hence \quad (\neg A[pUq])' &= \neg(A[pUq])' \qquad (9)
\end{aligned}$$

$$\begin{aligned}
(\neg A[pWq])' &= (E[\neg qU(\neg p \wedge \neg q)])' \\
&= E[(e_qt \wedge \neg q)U(\neg p \wedge \neg q)] \\
\neg(A[pWq])' &= \neg A[pW((e_act \wedge p) \vee q)] \\
&= E[\neg((e_act \wedge p) \vee q)U(\neg p \wedge \neg((e_act \wedge p) \vee q))] \\
&= E[((_act \wedge \neg q) \vee (\neg p \wedge \neg q))U(\neg p \wedge \neg q)] \\
&= E[(e_qt \wedge \neg q)U(\neg p \wedge \neg q)] \\
Hence \quad (\neg A[pWq])' &= \neg(A[pWq])' \qquad (10)
\end{aligned}$$

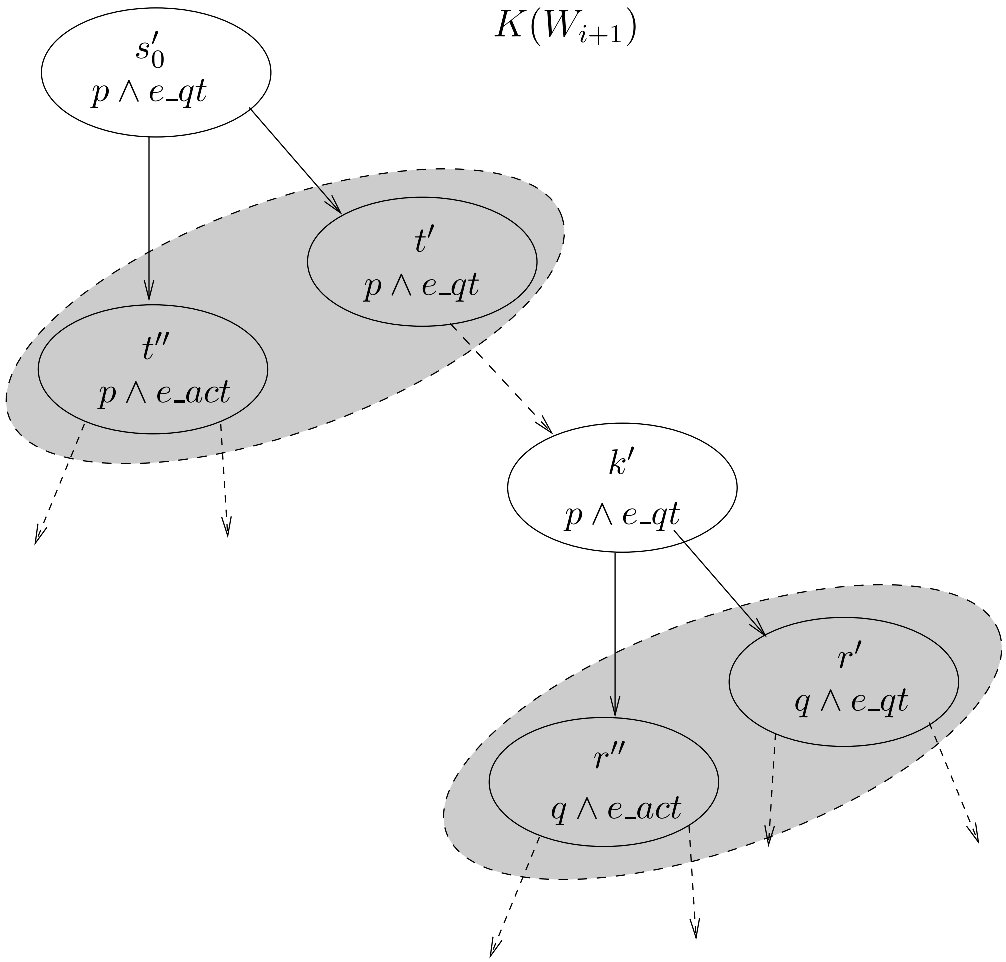

Fig. 1. $K(W_{i+1}, s' \models A[pU((e\_act \wedge p) \vee q)]$ t' and t'' enriches $t \in K(W_i)$ and t' simulates t, r' and r'' enriches $r \in K(W_i)$ and r' simulates t.