

Measurement of Continuous-Time $\Sigma\Delta$ Modulators: Implications of Using Spectrum Analyzer

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Abstract—In this paper, The difference between getting the output spectrum directly using spectrum analyzer and obtaining the spectrum digitally in measuring clock jitter effect on continuous-time $\Sigma\Delta$ modulator is analyzed. It is shown that clock jitter can be seen as input-referred or output-referred, depending on the nature of the measurement tool. Quantization noise and jitter noise are analyzed and compared graphically using a simple approach. The presented analysis is verified with system-level simulation of a 4th order bandpass continuous-time $\Sigma\Delta$ modulator.

I. INTRODUCTION

The continuously growing demand for higher data-rates communications accompanied by the demand for decreasing power consumption increases the required specifications from the Analog-to-Digital Converters (ADCs). Continuous-Time Sigma-Delta (CT $\Sigma\Delta$) modulators are receiving more and more attention due to their advantages compared to Discrete-Time (DT) $\Sigma\Delta$ modulators. Inherent anti-aliasing filtering, lower thermal noise, higher sampling rate and lower power consumption are all attractive advantages of CT $\Sigma\Delta$ modulators that make them interesting solutions for high data-rate wireless communication systems [1], [2].

Despite all their merits, CT $\Sigma\Delta$ modulators suffer from their sensitivity to clock jitter. The clock jitter noise of the feedback Digital-to-Analog Converter (DAC) is not shaped by the loop filter, due to its direct connection to the input node. It appears as a white noise in the signal band, and causes a degradation in the Signal-to-Noise-Ratio (SNR) of the modulator [3], [4]. There are many publications that discuss the clock jitter in CT $\Sigma\Delta$ modulators. Some of these publications analyze the clock jitter effect on CT $\Sigma\Delta$ modulators as in [3], [4], suggest solutions to decrease the clock jitter effect as in [5], [6], or present jitter simulations techniques as in [7], [8].

The traditional method to measure the performance of CT $\Sigma\Delta$ modulators is to apply the modulator output signal directly to a spectrum analyzer, see the output spectrum, and measure the in-band noise. Despite its simplicity, this method is misleading as it can not detect the SNR degradation due clock jitter effect [9]. The correct method to measure the modulator SNR, is to capture its output as a digital stream, apply Fourier Transform in digital domain, and extract the in-band noise from the calculated output spectrum. The difference between the two methods was discussed in few publications without sufficient explanation [10], [11]. In this paper, the difference between treating the output CT $\Sigma\Delta$ modulator as

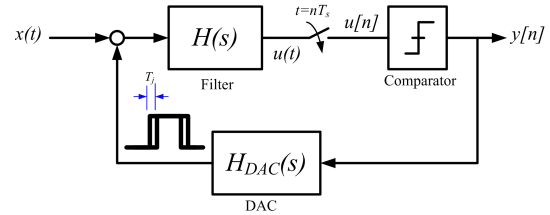


Fig. 1. Continuous-Time $\Sigma\Delta$ modulator.

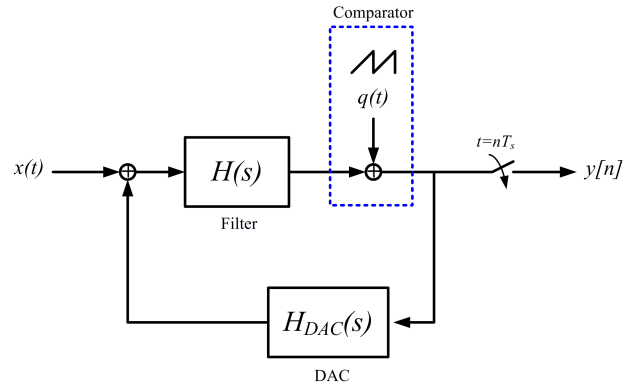


Fig. 2. Linear model of CT $\Sigma\Delta$ modulator.

analog signal or a digital bit stream is discussed, and the error due to using the traditional measurement method is explained.

II. MODULATOR LINEAR MODEL

CT $\Sigma\Delta$ modulator consists of three main blocks: loop filter, quantizer, and feedback DAC as shown in Fig. 1. The exact mathematical analysis of $\Sigma\Delta$ modulators is very difficult, due to non-linearity of the quantizer. A simple and widely-used approach to overcome this difficulty is to model the quantizer as an adder, and model the quantization noise as an additive noise source [12], as shown in Fig. 2.

By following the same modeling approach, the clock jitter effect, which causes the quantizer output transitions to deviate from the ideal positions, can be modeled as another noise source, as shown in Fig. 3. The new noise source represents jitter effect, i.e. the difference between quantizer output with jitter and the ideal output without jitter [7], [8]. Although it is widely adopted to add the jitter noise source to the DAC output, we believe that it makes more sense to add it to the quantizer output, because the quantizer is the block connected

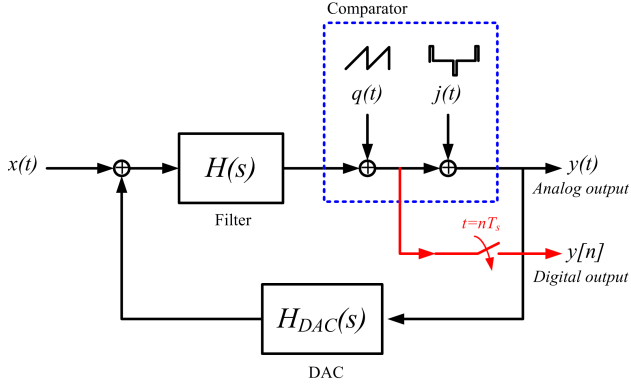


Fig. 3. Adding jitter effect to the linear model

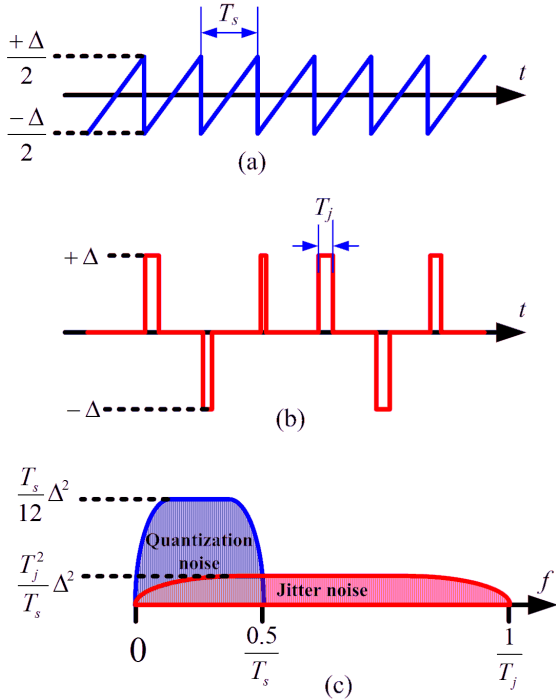


Fig. 4. Additive noise spectrum (a) Quantization noise waveform. (b) Jitter noise waveform. (c) Spectrum of quantization noise and jitter noise.

directly to the clock. So, the clock jitter affects the quantizer, which passes this effect to the DAC.

A. Quantization noise

Quantization noise is signal-dependent, but for the sake of simplicity it can be approximated as a sawtooth signal bounded between the two possible extremes of the quantization error $\pm\Delta/2$ [13], as shown in Fig. 4(a), where Δ is the quantization step. Based on this approximation, the quantization noise power is calculated to be:

$$P_Q = \Delta^2/12 \quad (1)$$

The quantization noise power can be assumed uniformly distributed in the Nyquist band, i.e. from 0 to $f_s/2$, as shown in Fig. 4(c).

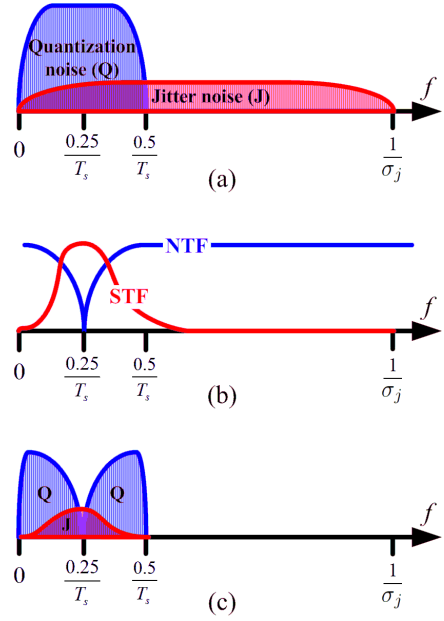


Fig. 5. Noise shaping in case of digital output (a) Quantization and jitter noise before shaping. (b) NTF and STF of the modulator. (c) Noise shaping seen with digital output.

B. Jitter noise

Jitter noise is a train of pulses which have a variable width depending on clock jitter, as shown in Fig. 4(b). The height of the jitter noise pulses is equal to the quantization step, and the polarity of the pulse is signal-dependent. Jitter noise has a much wider bandwidth compared to quantization noise due to the thin width of the jitter noise pulses. Jitter noise power can be calculated to be [7]:

$$P_j = \frac{T_j}{T_s} \Delta^2 \quad (2)$$

where T_j is the rms clock jitter, and T_s is the sampling time. The jitter noise spectrum can be assumed uniform from 0 to $1/T_j$ (the approximate bandwidth of the rectangular pulse of width T_j), as shown in Fig. 4(c).

III. NOISE SHAPING

The noise spectrum shown in Fig. 5(a) represents the noise added by the quantizer before shaping. The shaping of the noise is dependent on the point where it was injected. For output-referred noise, the noise is shaped by the Noise-Transfer-Function (NTF), which usually has a notch in the band of interest as shown in Fig. 5(b), where a bandpass CT $\Sigma\Delta$ modulator centered at quarter the sampling frequency is taken as an example. For input-referred noise, the noise is shaped by the Signal-Transfer-Function (STF), which is usually maximum at the band of interest.

A. Digital output

In normal operation of the modulator, the output is taken as a digital bit stream. Referring to Fig. 3, it can be seen that the quantization noise is NTF shaped, and the jitter noise is STF

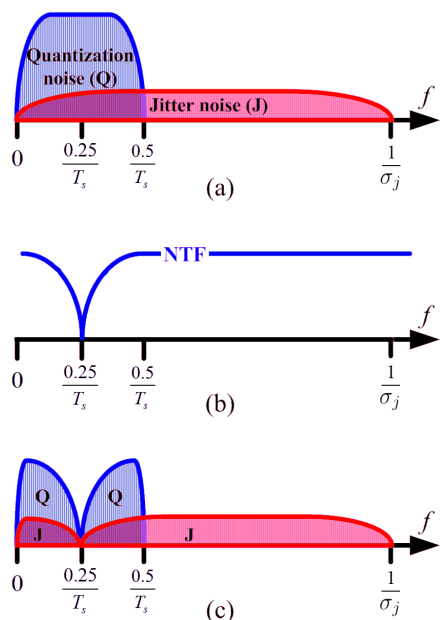


Fig. 6. Noise shaping in case of analog output (a) Quantization and jitter noise before shaping. (b) NTF of the modulator. (c) Noise shaping seen with spectrum analyzer.

shaped, as shown in Fig. 5. Although the quantization noise is very small in the band of interest, the band is filled with jitter noise, and the SNR of the modulator is limited by the clock jitter.

B. Analog output

If the output of the modulator is taken as an analog signal, as in modulator measurement using spectrum analyzer, both the quantization noise, and the jitter noise are NTF shaped, as shown in Fig. 6. In this case, the jitter disappears from the band of interest, and the “measured” SNR is only limited by the quantization noise.

IV. VALIDATION

To validate the analysis, a 4th order single-bit bandpass CT $\Sigma\Delta$ modulator was designed and simulated with the presence of clock jitter. The output of the modulator was extracted twice using two different methods:

Digital spectrum method: This is the normal operation method, in which the output is treated as a digital bit stream, and there is only one output bit for each clock cycle. The spectrum of the output taken this way is referred to as “digital spectrum”.

Analog spectrum method: In this method, the modulator output is treated as an analog signal and re-sampled using a clean clock with a very high sampling rate relative to the modulator sampling rate. The spectrum of the output taken this way is referred to as “analog spectrum”.

Both the digital spectrum and the analog spectrum are shown in Fig. 7. The digital spectrum is limited to half the modulator sampling frequency, while the analog spectrum extends much wider depending on the re-sampling clean clock

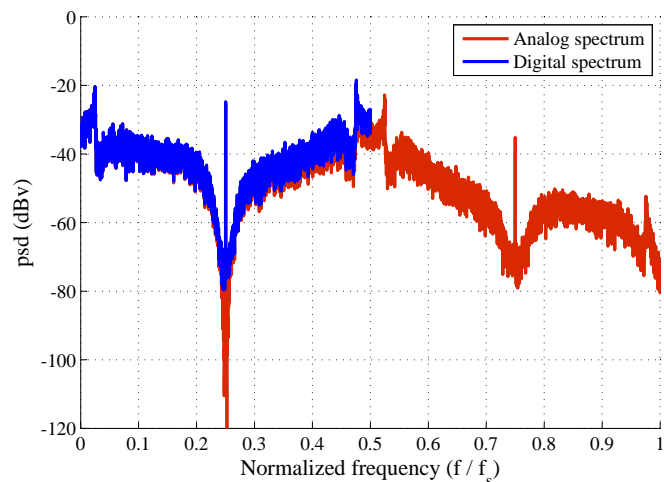


Fig. 7. Simulated output spectrum of the modulator with both analog and digital spectrum.

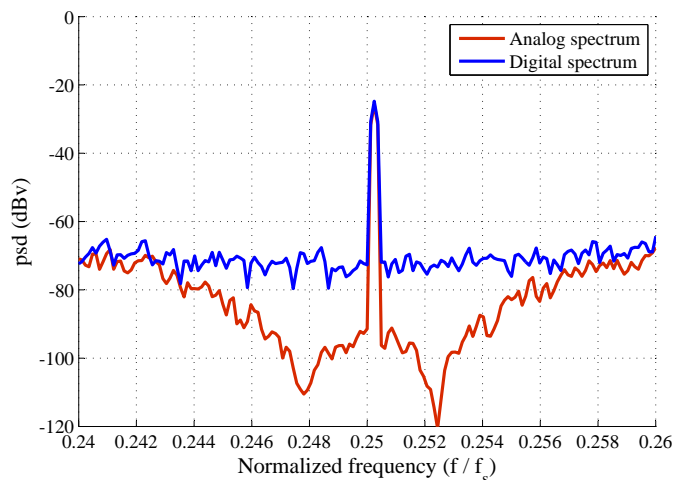


Fig. 8. Simulated output spectrum with zoom on the signal band.

frequency. Only a part of the analog spectrum is shown for better viewing and comparison with the digital spectrum. To see the band of interest more clearly, the digital spectrum and the analog spectrum are shown again in Fig. 8, where only a narrow band around the NTF notch is shown. It can be seen from the figure that the analog spectrum is very clean and has no jitter noise, while the digital spectrum is filled with jitter noise.

For more validation, the simulation was re-run using different values of the rms clock jitter. For each run, the analog spectrum was extracted as shown in Fig. 9. It can be seen in the figure that the jitter noise disappears only from the first Nyquist band, but it appears in the notch replica. This is due to wide bandwidth of the jitter noise compared to the quantization noise which is approximately band-limited as was shown in Fig. 5.

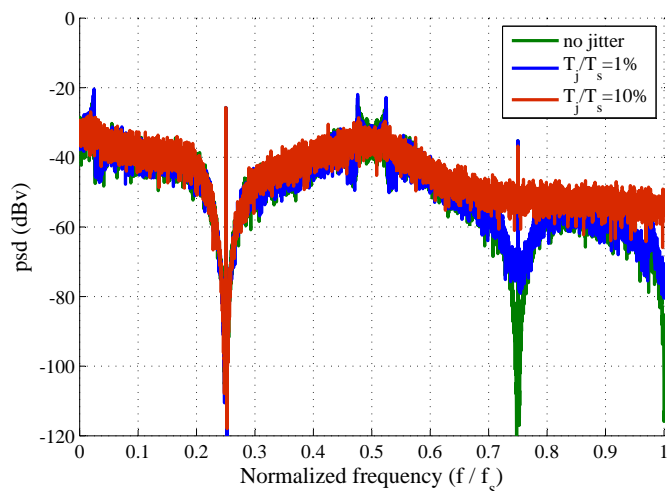


Fig. 9. Simulated output spectrum for different values of jitter.

V. CONCLUSION

The output spectrum of the CT $\Sigma\Delta$ modulator was shown to be dependent on the way of treating the modulator output. Jitter noise can be seen as input-referred or output-referred according to the measurement tool nature. In case of obtaining the output digitally, the jitter noise is STF shaped, but in case of using a spectrum analyzer, the jitter noise is NTF shaped, and the measured SNR is incorrect. Simulation results showed good agreement with presented analysis.

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