

Time based quantizers

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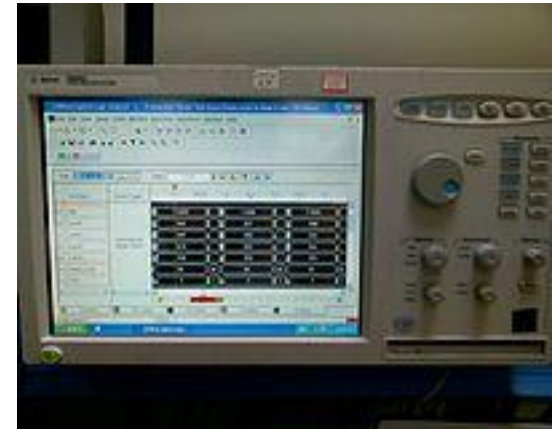
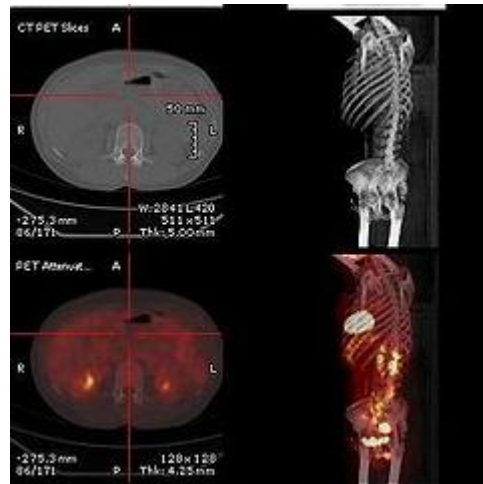
LIP6 Laboratory



Time to digital Converters (TDC)

Applications

- Commercial time-of-flight applications such as Laser range-finding
- Positive electron tomography medical imaging technology
- Logic Analyzers



Circuit & Systems

A fundamental element in systems made of closed loop integrated circuits that needs precise control and alignment of timing signals such as :

- Phase Locked Loop (PLL)
- Delay Locked Loop (DLL)
- Clock Data Recovery (CDR)

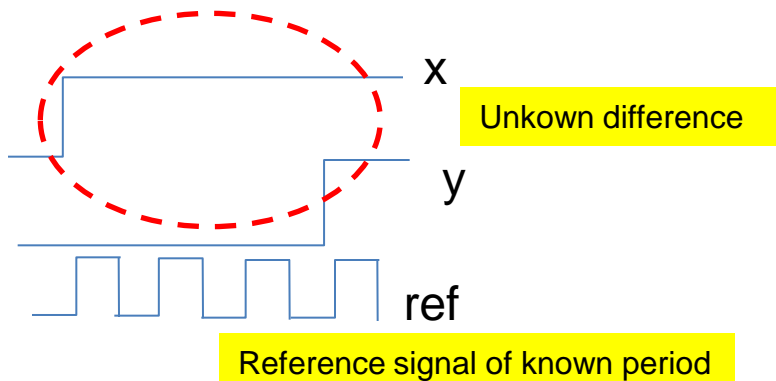
Types of Time to Digital Converters

1) Classical TDC

Quantizing the difference in time between 2 signals

(PLL, DLL, CDR)

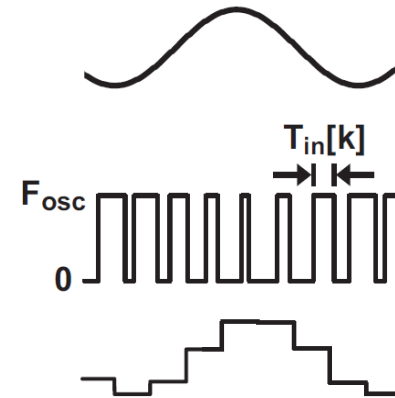
Quantizing the difference between x and y



2) TDC as analog signal quantizers

A replacement for traditional voltage quantizers

(High resolution wideband ADC)

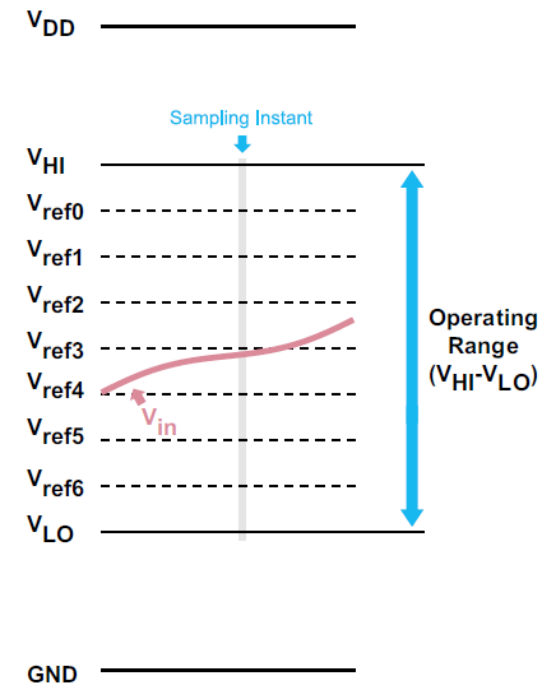
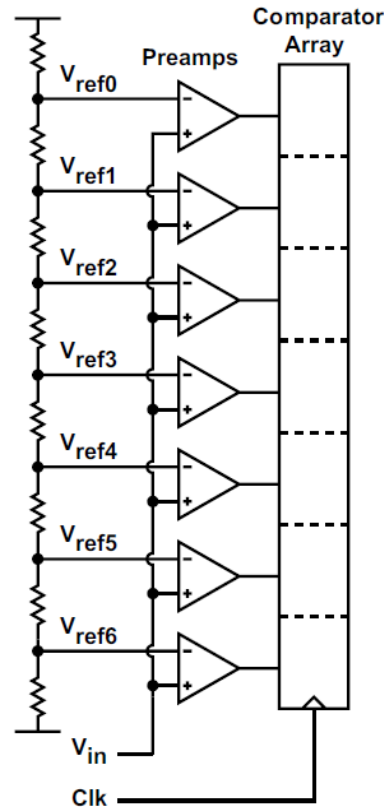


Technology Scaling Challenges

Multi-bit quantization using conventional Flash voltage quantizer

Reduced supply & dimensions

- Higher F_t ✓
- Lower V_{ref} accuracy ✗
- Metastability ✗



Motivation for TDC as ADC

Detecting an edge transition from gnd to Vdd is easier than a voltage step of $V_{dd}/(2^N)$

- Only Vdd and gnd are used (low supply compatible)
- High precision detecting transitions (Resolution)
- Mostly digital implementations benefits of tech scaling in terms of power and area

Concept of TDC

The count of cycles of the reference signal represents the quantized value of T_{in}

$$T_{out}[k] = Out[k] T_q$$

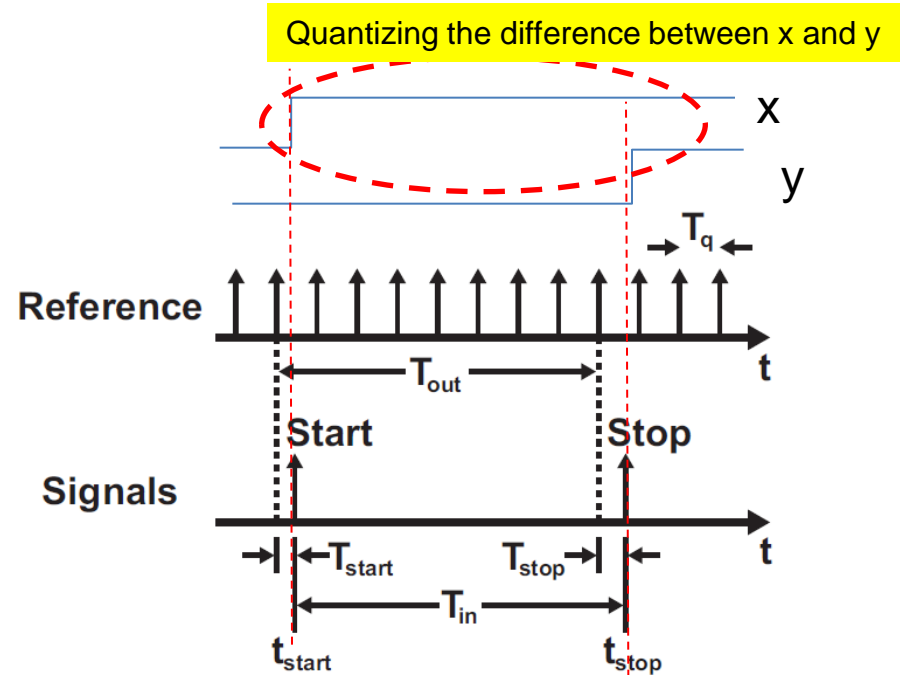
Quantized value of T_{in}

Count of T_q during T_{in}

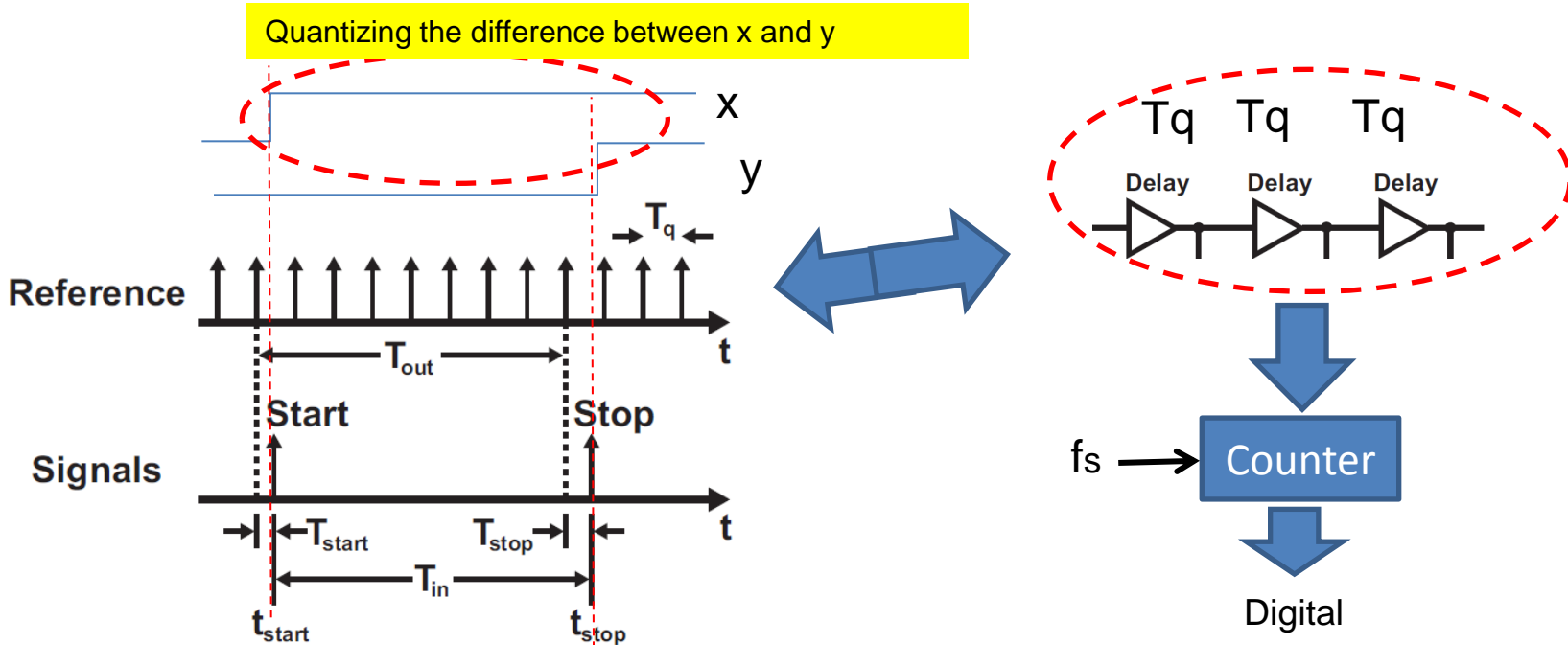
$$T_{error}[k] = T_{stop}[k] - T_{start}[k].$$

$$T_{out}[k] = T_{in}[k] - T_{error}[k],$$

$$Out[k] = \frac{T_{in}[k] - T_{error}[k]}{T_q}.$$



Concept of TDC



Definitions:

TDC resolution : T_q (reference signal period)

----- Limited by Technology Min gate delay

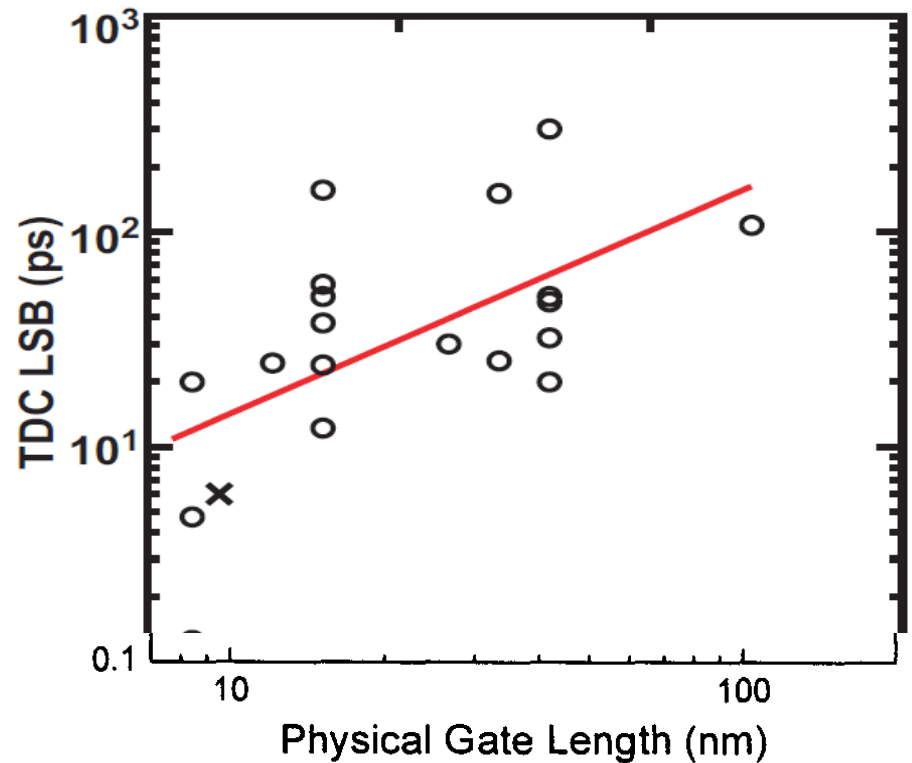
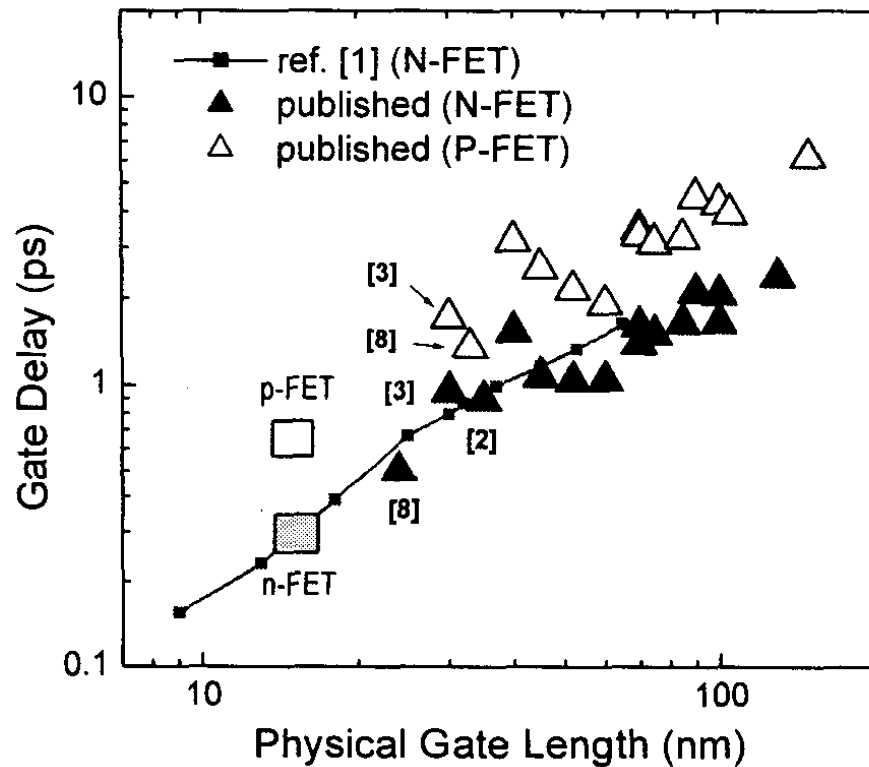
TDC Dynamic range: $\text{Max_count} * T_q$

----- Limited by f_s

To enhance the TDC resolution:

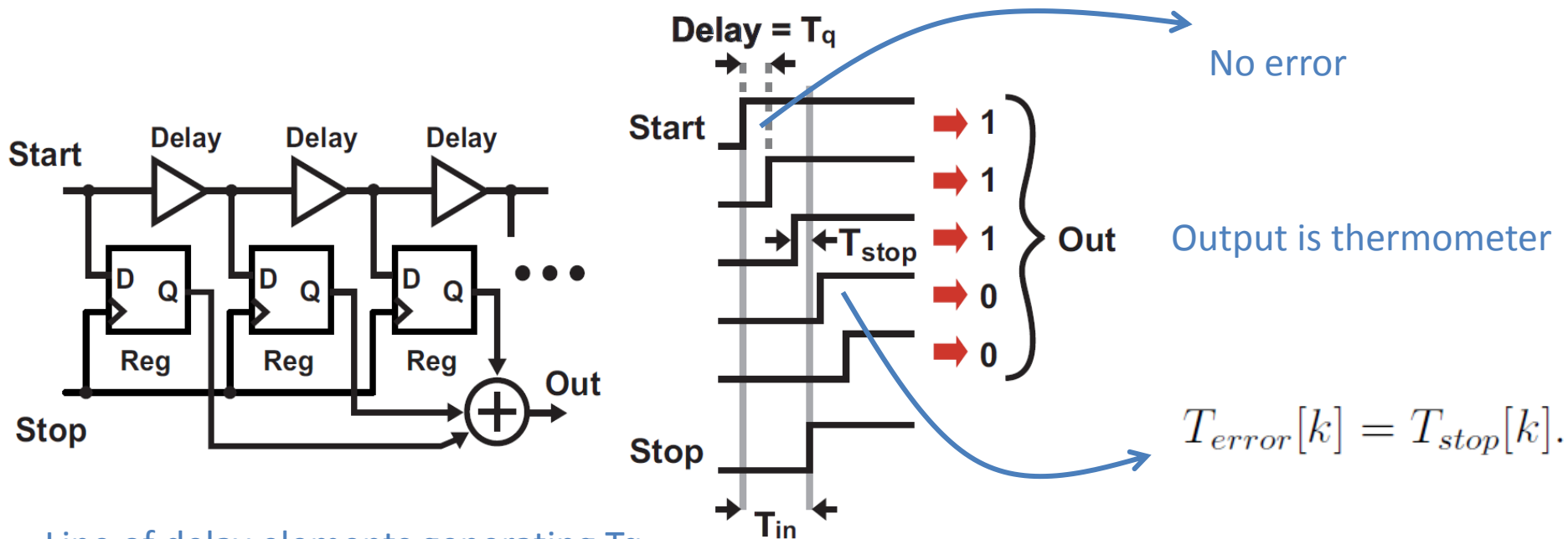
- 1) Technology advancement to lower Technology min gate delay
- 2) Special design techniques to go below Technology min gate delay

Technology Scaling and TDC resolution



Gate Delay decreases in new technologies and so is the TDC (LSB)

1a) Inverter chain based

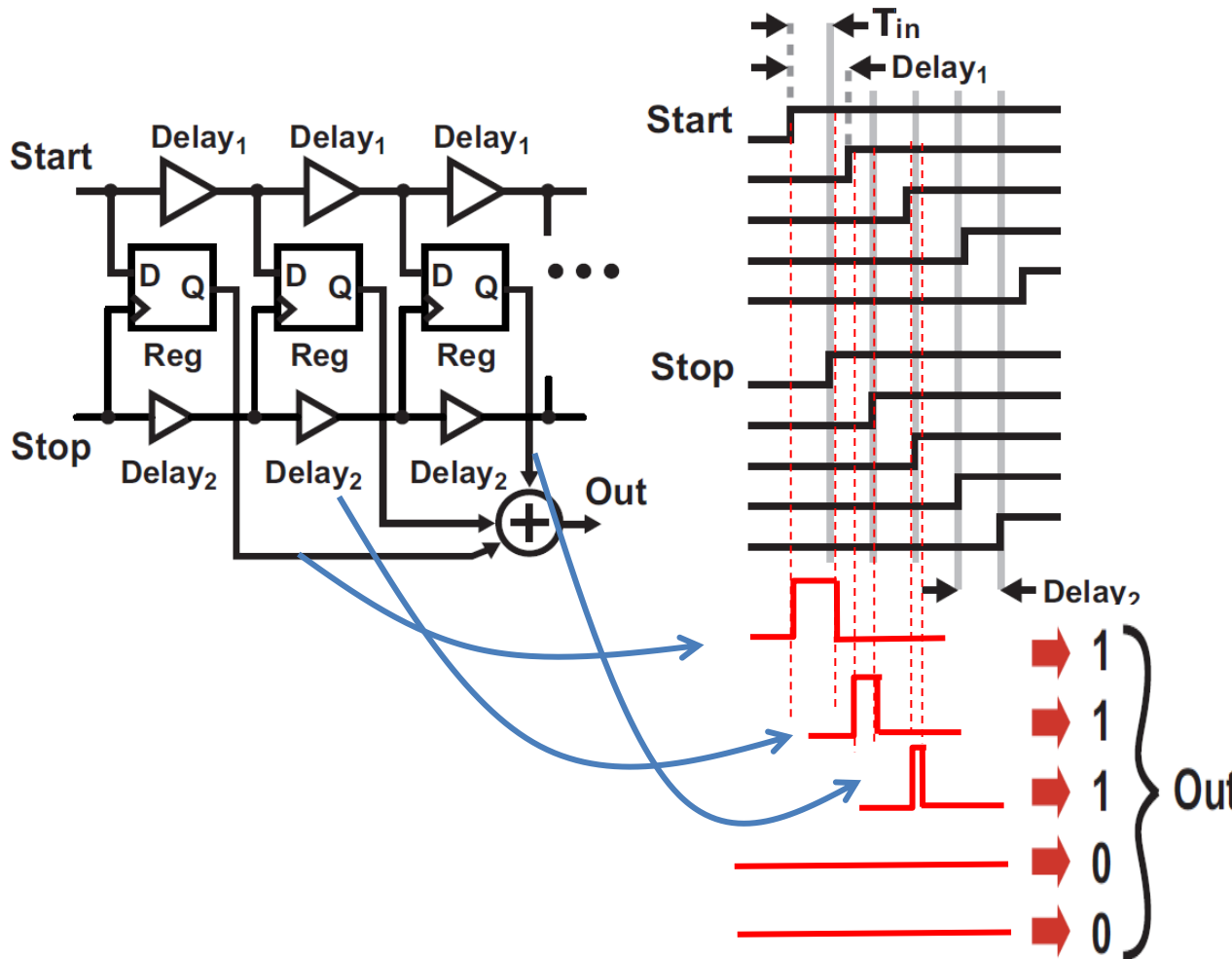


- Line of delay elements generating T_q
- T_q propagates through the line
- Registers clocked by the stop signal hold the final value of each delay (thermometer)
- Addition of the registers output gives the quantized value of T_{in}

For N-bit resolution, the number of delay elements = 2^N

- High Cost (area)
- High Power

1b) Vernier based ($T_{start} - T_{stop} < \text{delay}_1$)



delay₁ > delay₂
 $T_q = \text{delay}_1 - \text{delay}_2$

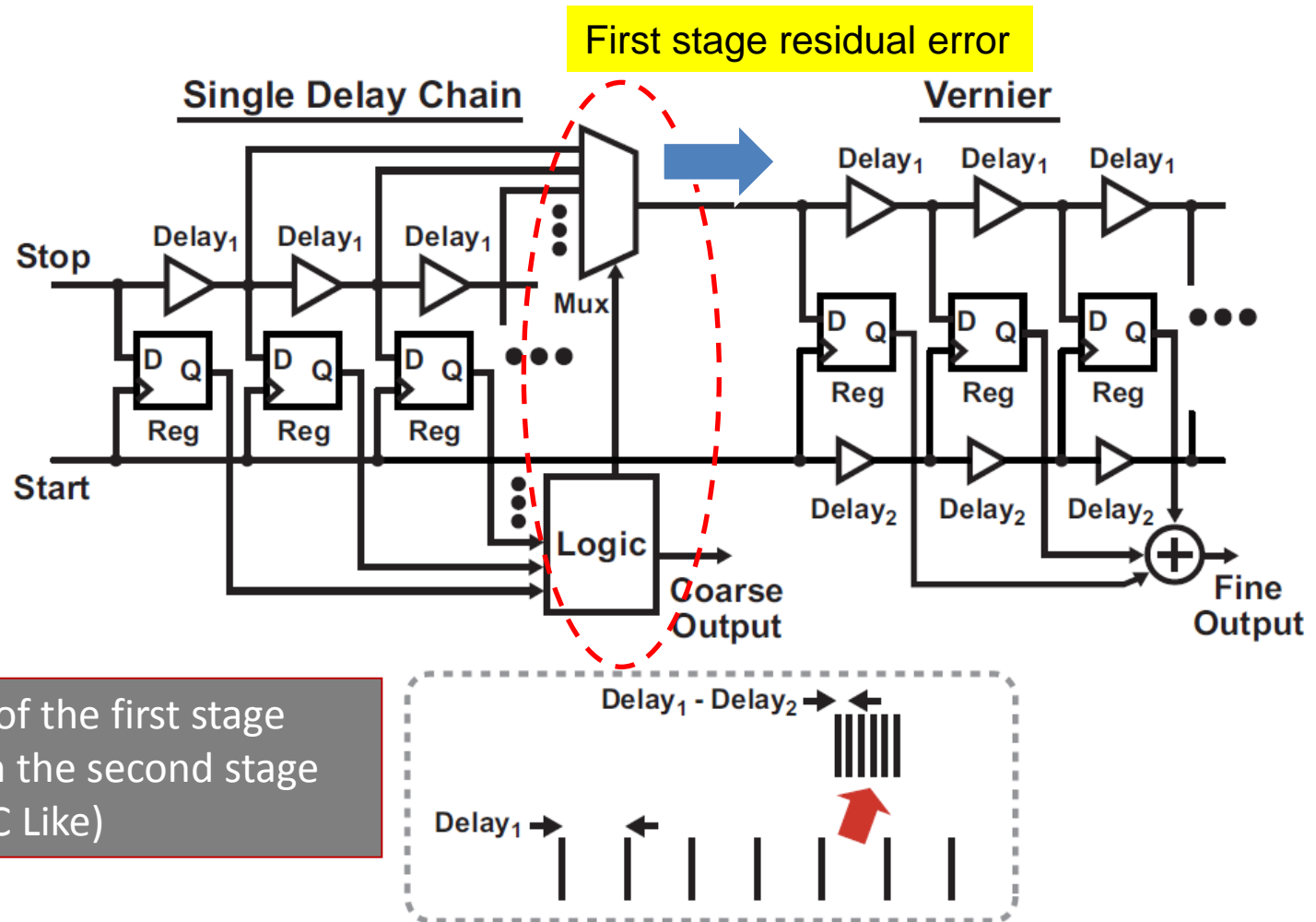
Output is thermometer

$$T_{error}[k] = T_{stop}[k]$$

Varying width pulse

- Better resolution at the same technology gate delay ✓
- Relatively large number of delay elements ✗

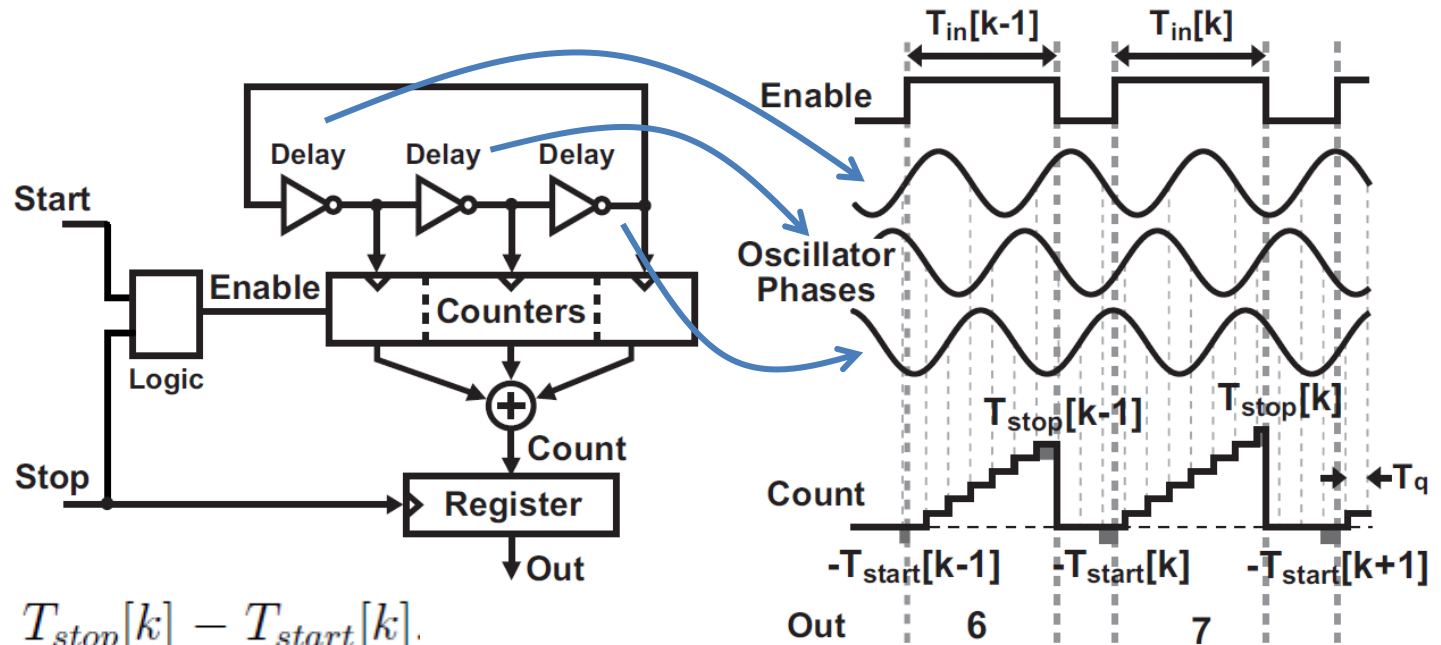
1c) Dual Step TDC



Amplifies the error of the first stage
And quantize it with the second stage
(Two-step Flash ADC Like)

Compromise Resolution and number of delay elements ✓

2a) Oscillator based TDC

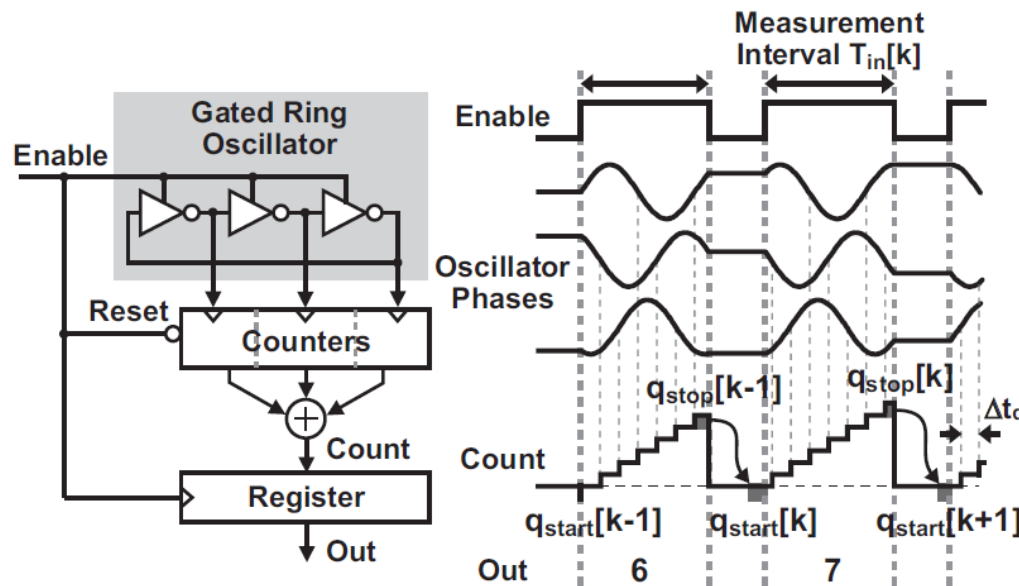


$$T_{error}[k] = T_{stop}[k] - T_{start}[k].$$

Least number of delay elements w.r.t preceding topologies ✓

Can we have better ?

2b) Gated Ring Oscillator based TDC



Theoretical approach !!
Not easy to implement

$$T_{start}[k] = T_{stop}[k-1]$$

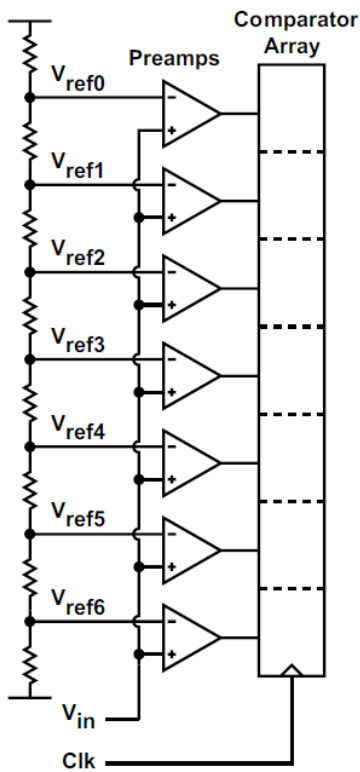
$$T_{error}[k] = T_{stop}[k] - T_{stop}[k-1]$$

Doubling OSR adds 9dB of SQNR
Like a first order SigmaDelta

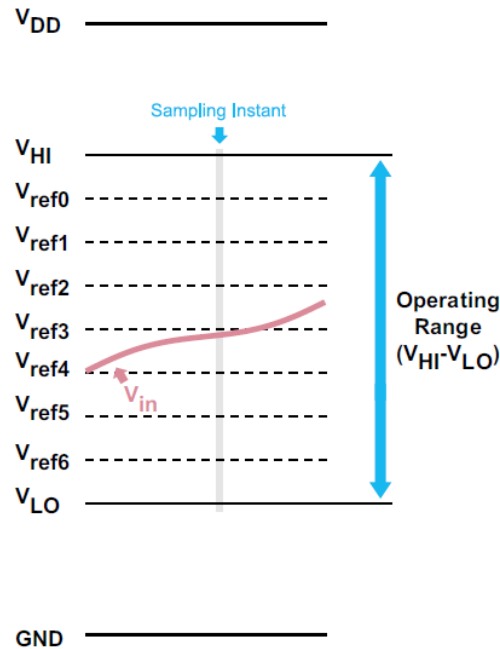
- First Order Noise Shaping for both Q and mismatch
- Quantization noise is white and performance then is enhanced by oversampling

Time domain quantizers in ADC

Multi-bit quantization using



(1) conventional Flash voltage quantizer



(2) Time Domain Quantizer



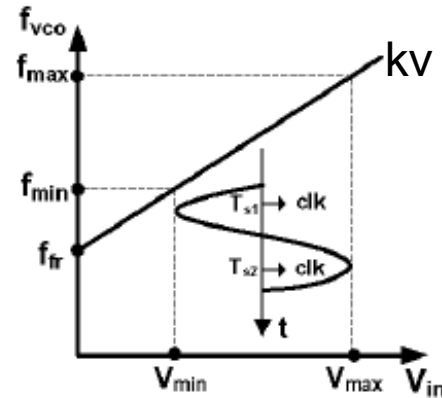
Motivation for TDC as ADC

Detecting an edge transition from gnd to Vdd is easier than a voltage step of $V_{dd}/(2^N)$

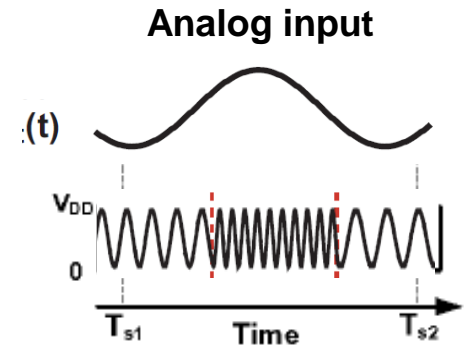
3) VCO-based ADC principle

May be seen as two Operations:

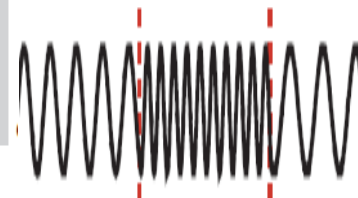
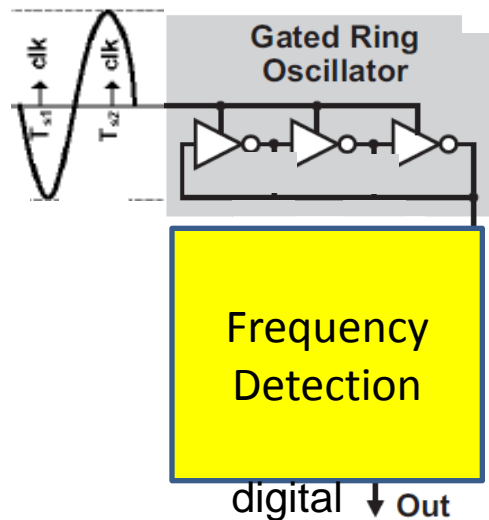
- Voltage to frequency (Time) conversion
- Time to digital conversion



(a) Voltage-to-time conversion



(b) VCO output in time domain

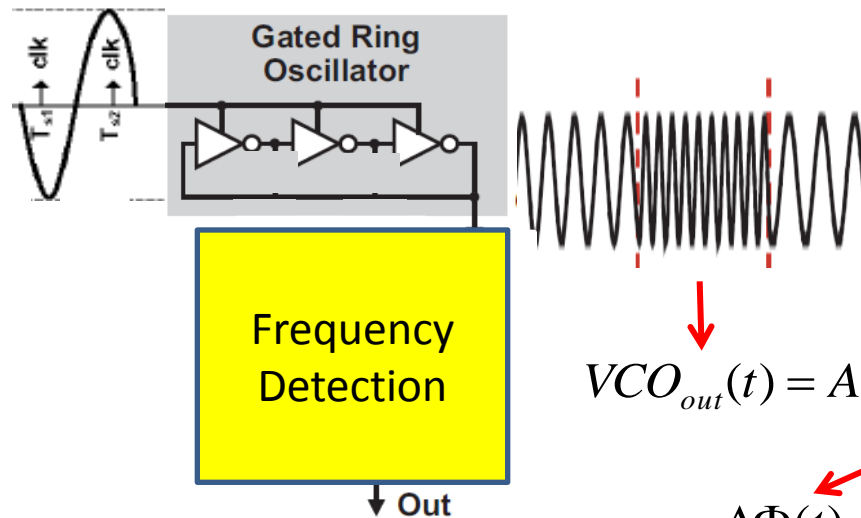


How ?

Differences with gated ring oscillator

- The enable signal replaced by analog continuous voltage to be converted
- The oscillator is always running since the input is continuously varying

3) VCO-based ADC principle



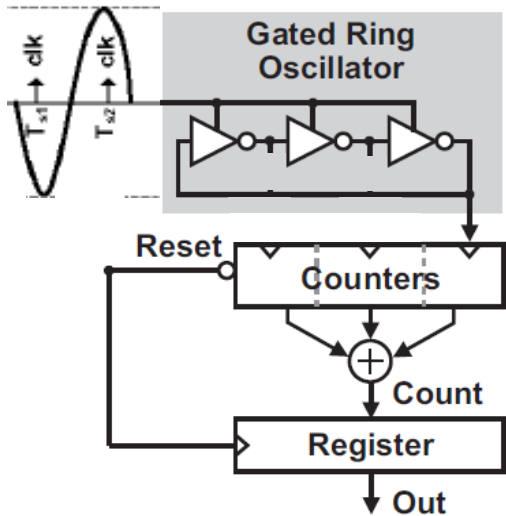
$$VCO_{out}(t) = A \sin\left(2\pi \int (K_v v_{tune}(t) + f_{fr}) dt\right)$$

$$\Delta\Phi(t) = 2\pi \int (K_v v_{tune}(t)) dt$$

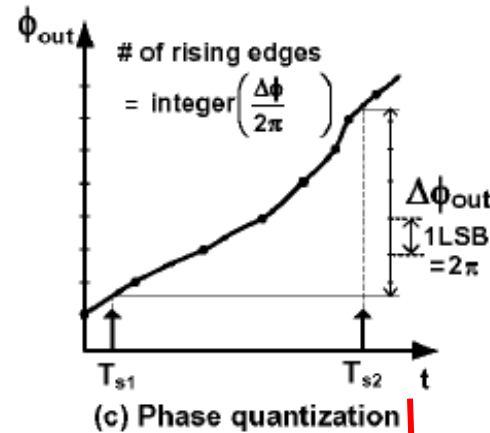
$$f_{VCO} = \frac{\Delta\Phi(t)}{\Delta t} = \frac{d}{dt} \left(2\pi \int (K_v v_{tune}(t)) dt \right)$$

How to implement this ?

3) VCO-based ADC operation



n : number of rising edges



# of rising edges	Digital output
⋮	⋮
5	0101
6	0110
7	0111
⋮	⋮

(d) Digital code generation

Quantization step (LSB) = 2π

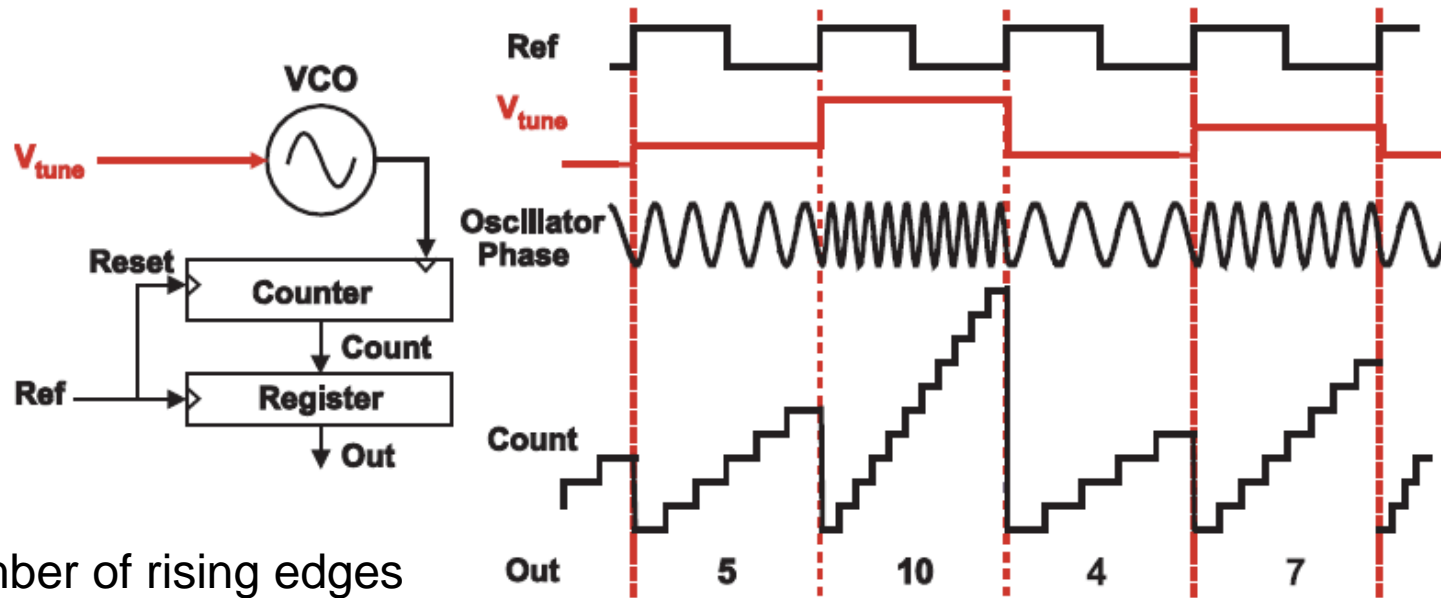
$$\Delta\Phi(t) = 2\pi * n$$

$$f_{VCO} = \frac{\Delta\Phi(t)}{\Delta t} = \frac{\Delta\Phi(t)}{T_S} = \Delta\Phi(t) f_S$$

Quantized

$$f_{VCO} = 2\pi * f_S * n$$

3a) Single phase VCO-based Quantizer



N: number of rising edges

$$f_{VCO} = K * n$$

$$\text{Quantization step (LSB)} = 2\pi$$

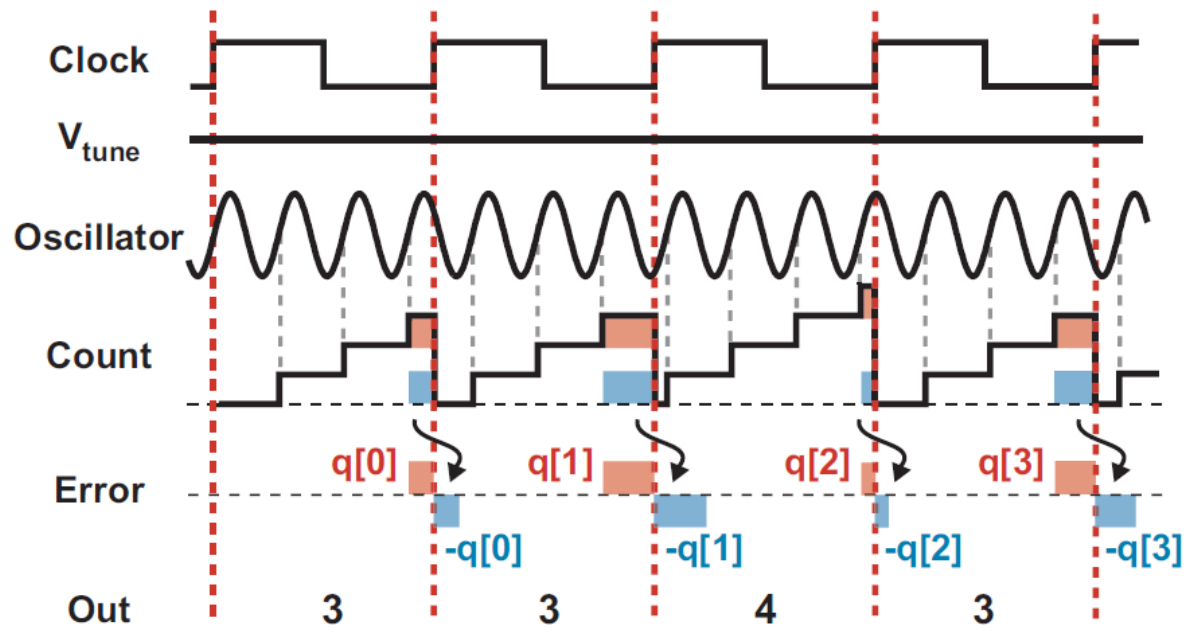
$$f_{VCO} = 0, f_s, 2 * f_s, \dots, (2^N - 1) * f_s$$

For high resolution, f_{VCO} gets very high to be feasible

n : quantizer resolution

Noise shaping of VCO-based Quantizer

Constant input and Output is toggling due to residual error memorization

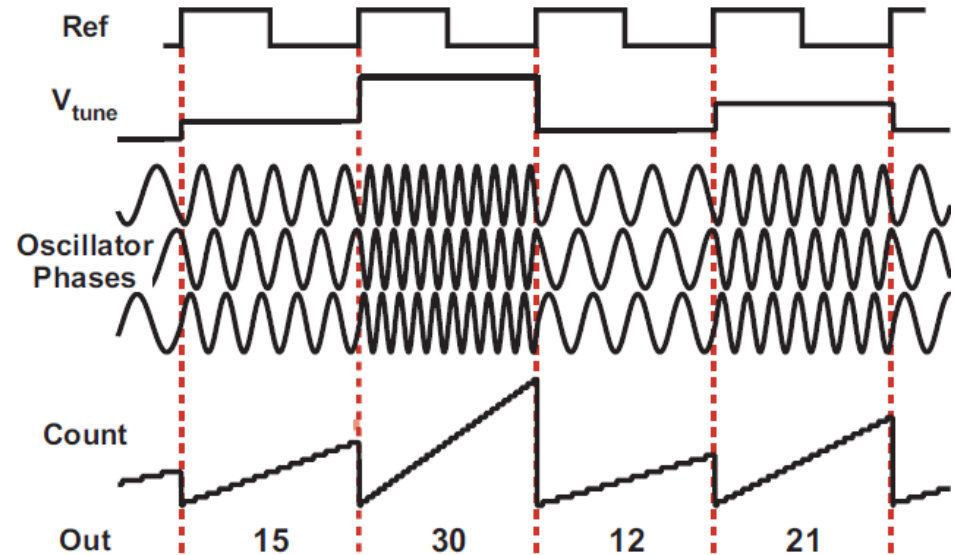
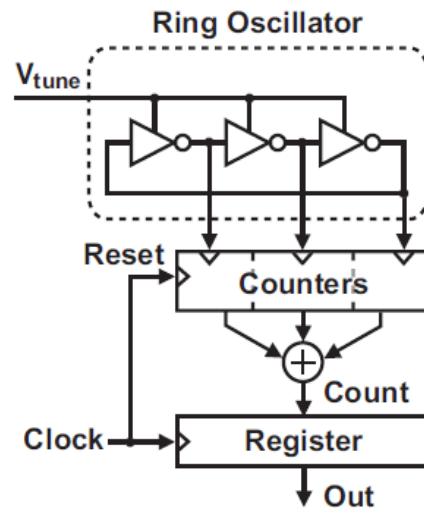


First-order noise shaping of a classical VCO-based ADC

$$Error[k] = q[k] - q[k - 1],$$

Doubling OSR adds 9dB of SQNR
Like a first order Sigma Delta Modulator (SDM°)

3b) Multi-phase VCO-based Quantizer



$$f_{VCO} = 0, \frac{f_s}{m}, \frac{2 * f_s}{m}, \dots, \frac{(2^N - 1)}{m} * f_s$$

n : quantizer resolution
 m : inverter stages

$$f_{VCO} = K_2 * n \quad K_2 = (2\pi / m) * f_s$$

Quantization step (LSB) = $(2\pi / m) * f_s$

Selecting $m = 2^N - 1 \rightarrow f_{VCO_max} = f_s$

3b) Multi-phase VCO-based Quantizer

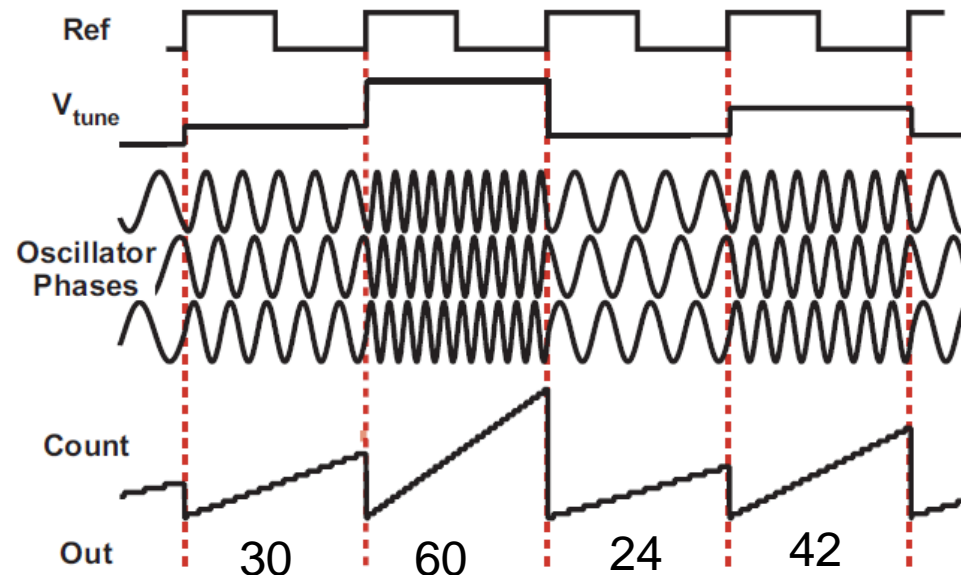
$$f_{vco} = \frac{0xfs}{2(2^N - 1)}, \frac{1xfs}{2(2^N - 1)}, \dots, \frac{(2^N - 1)xfs}{2(2^N - 1)}$$

$$\text{No of inverters} = 2^N - 1$$

$$f_{vco}(\text{max}) = f_s / 2$$

Further decrease f_{vco} for the same resolution

Counting rising and falling edges

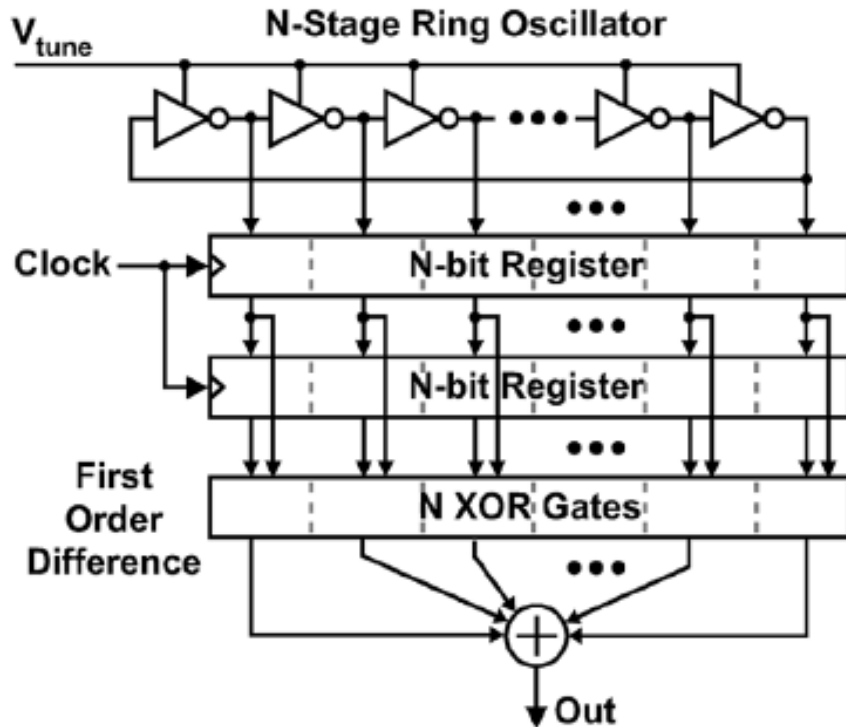


Drawbacks

- 1) Reset pulse can coincide with Ring Clock Pulse (Asynchronous with Reset), Noise shaping will vanish
- 2) Complex implementation for higher OSR and more quantization levels (counters)

3c) Quantizer Efficient implementation

Eliminating the Counters reset problem



$$f_{vco} = \frac{\Delta\Phi(t)}{\Delta t} = \frac{d}{dt} \Phi(t)$$

Frequency Detection

Number of inverters switching state between two sampling instances is a count of the zero crossings during this time

Example

```

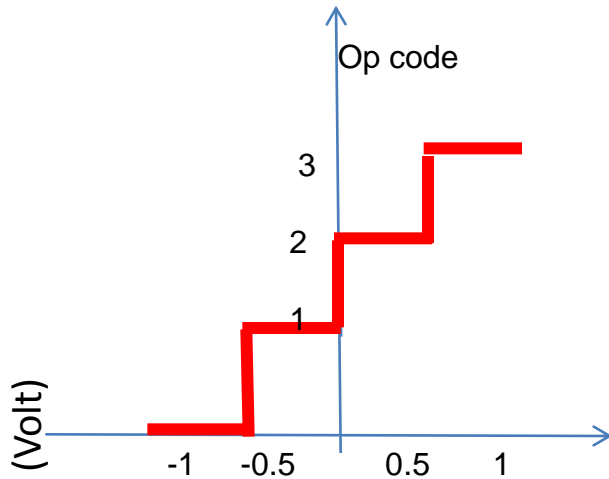
1 0 1 0 1 0 0 1 0
XOR
0 1 1 0 1 0 1 0 1

```

1 1 0 0 0 0 1 1 1

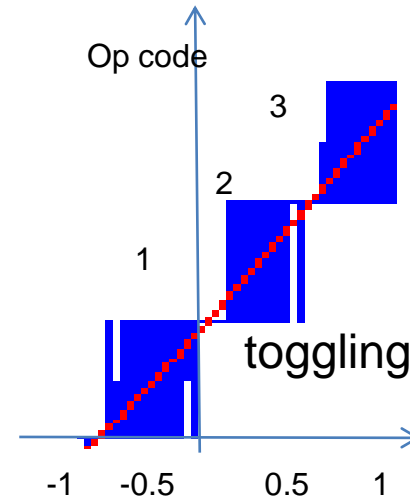
5 transitions detected by XOR

VCO-based Quantizer toggling



2-bit Flash Quantizer

V_{in}	o/p code
-1 to -0.5	0
-0.5 to 0	1
0 to 0.5	2
0.5 to 1	3

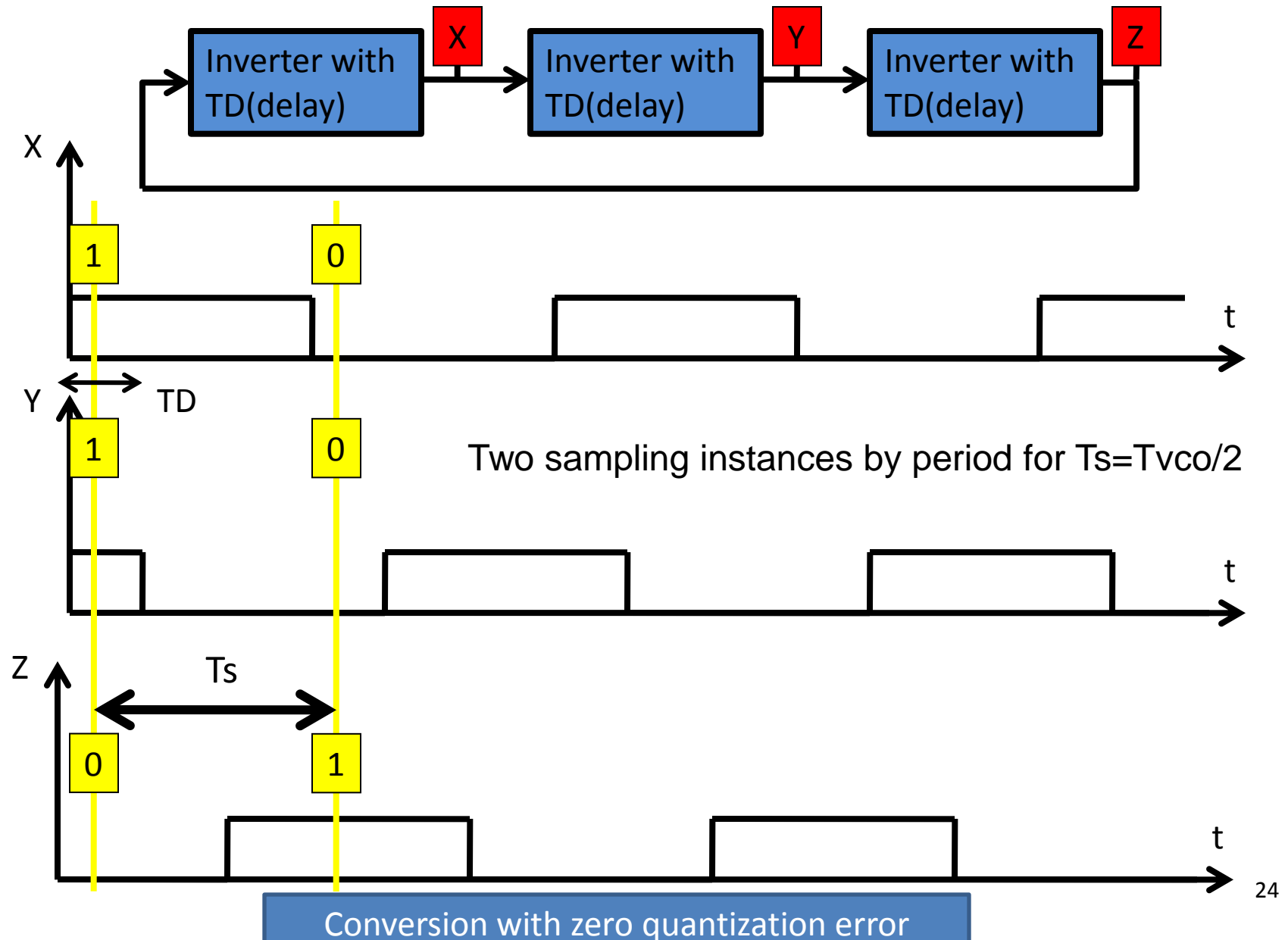


2-bit VCO Quantizer

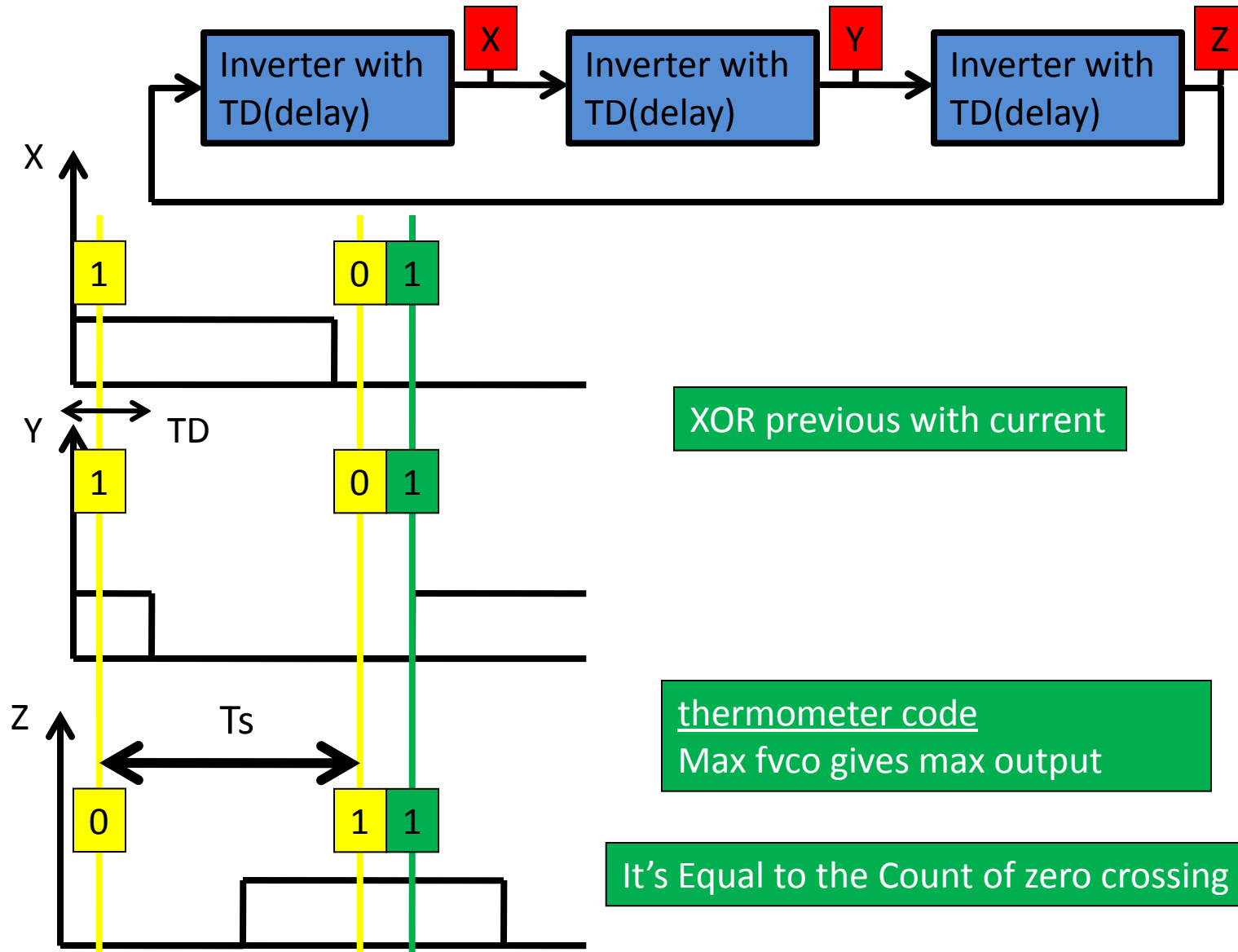
V_{tune}	F_{vco}	o/p(code)
-1	0	0
-1/3	$F_s/6$	1
1/3	$F_s/3$	2
1	$F_s/2$	3

Frequencies ($k \cdot f_s / 2m$) have zero quantization error

Ex1: Quantizing $V_{tune}=1V$ ($f_{vco}=f_s/2$)



Ex1: Quantizing $V_{tune}=1V$ ($f_{vco}=f_s/2$)



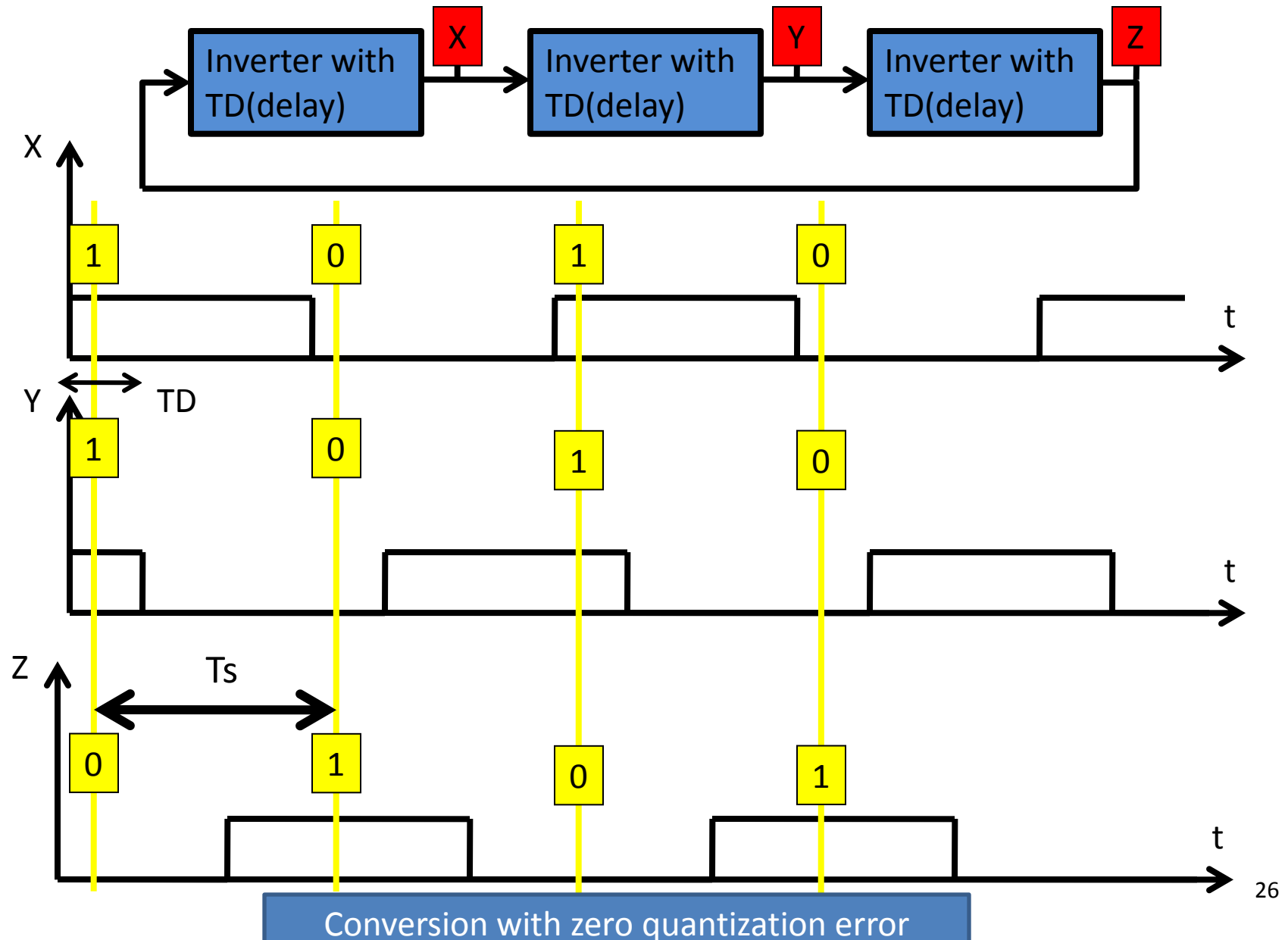
XOR previous with current

thermometer code
Max f_{vco} gives max output

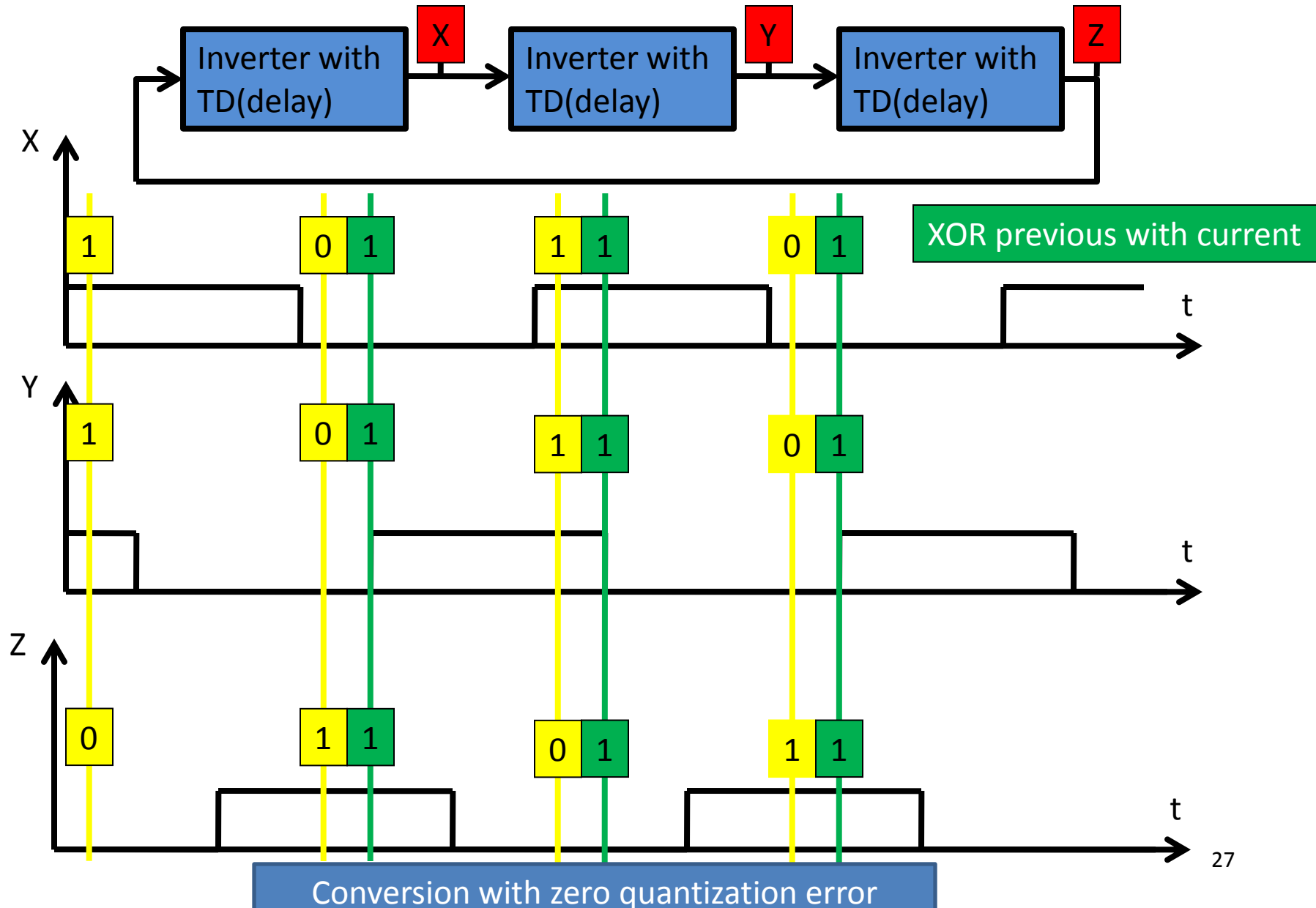
It's Equal to the Count of zero crossing

Conversion with zero quantization error

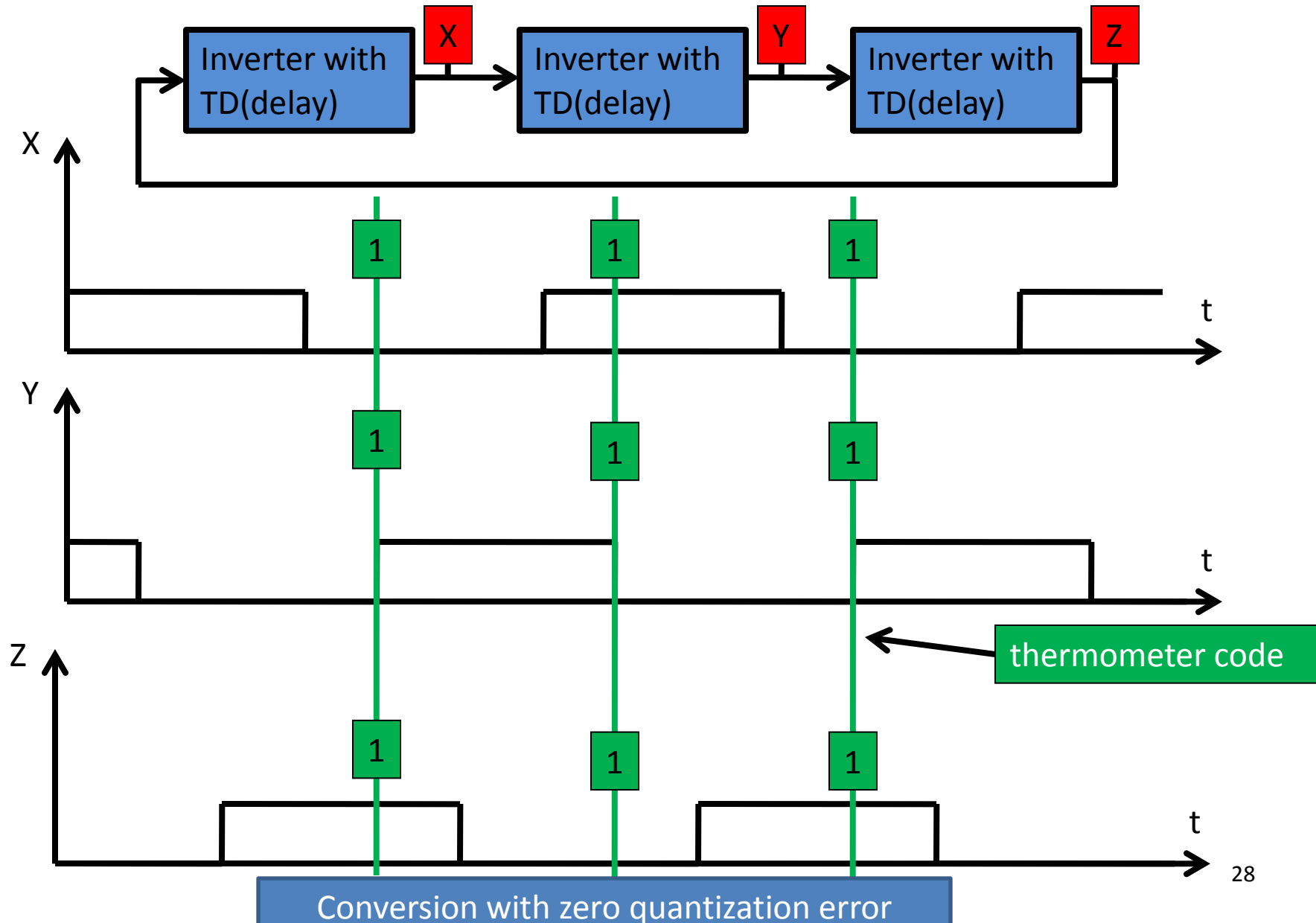
Ex1: Quantizing $V_{tune}=1V$ ($f_{vco}=f_s/2$)



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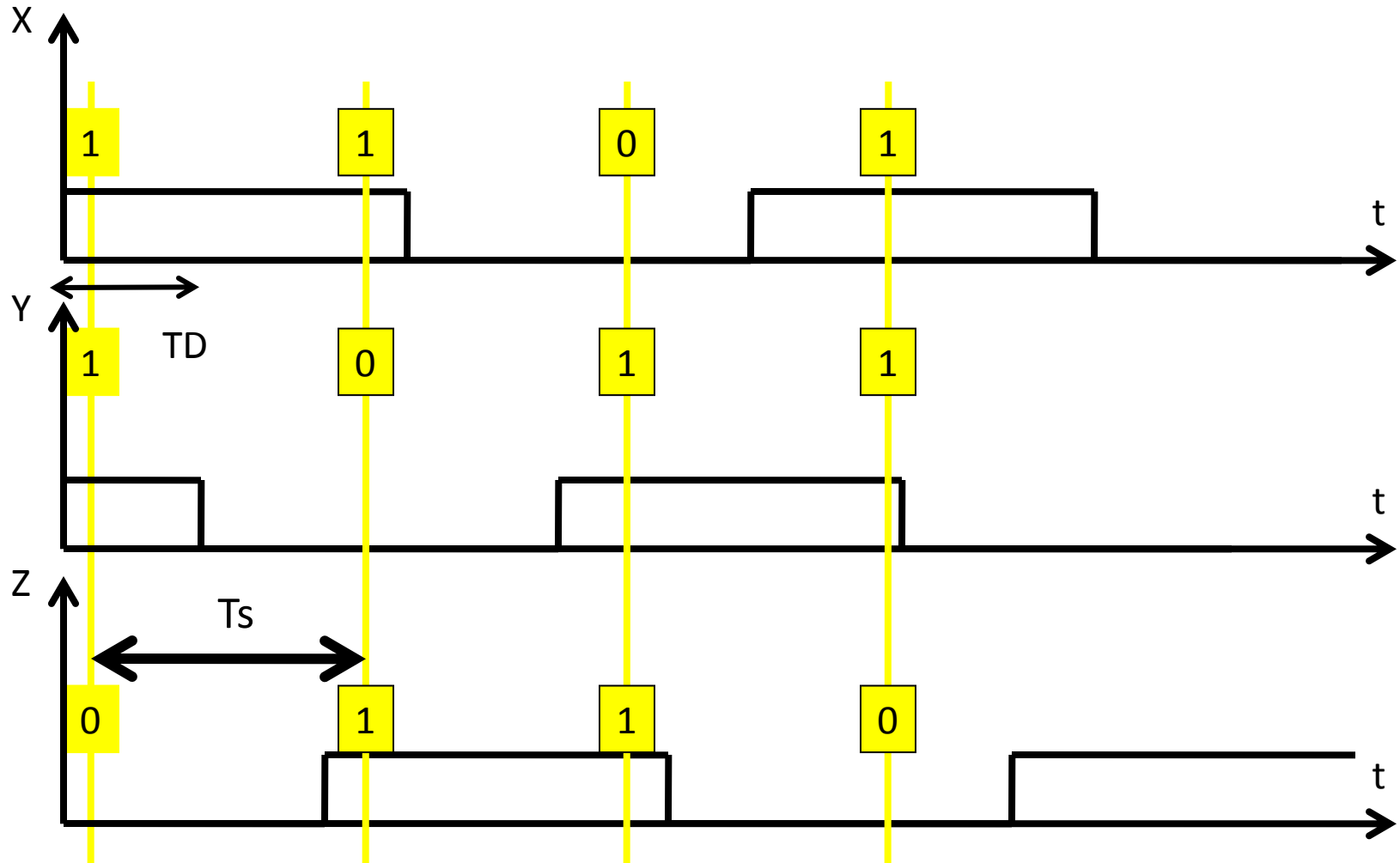


Ex1: Quantizing $V_{tune}=1V$ ($f_{vco}=f_s/2$)



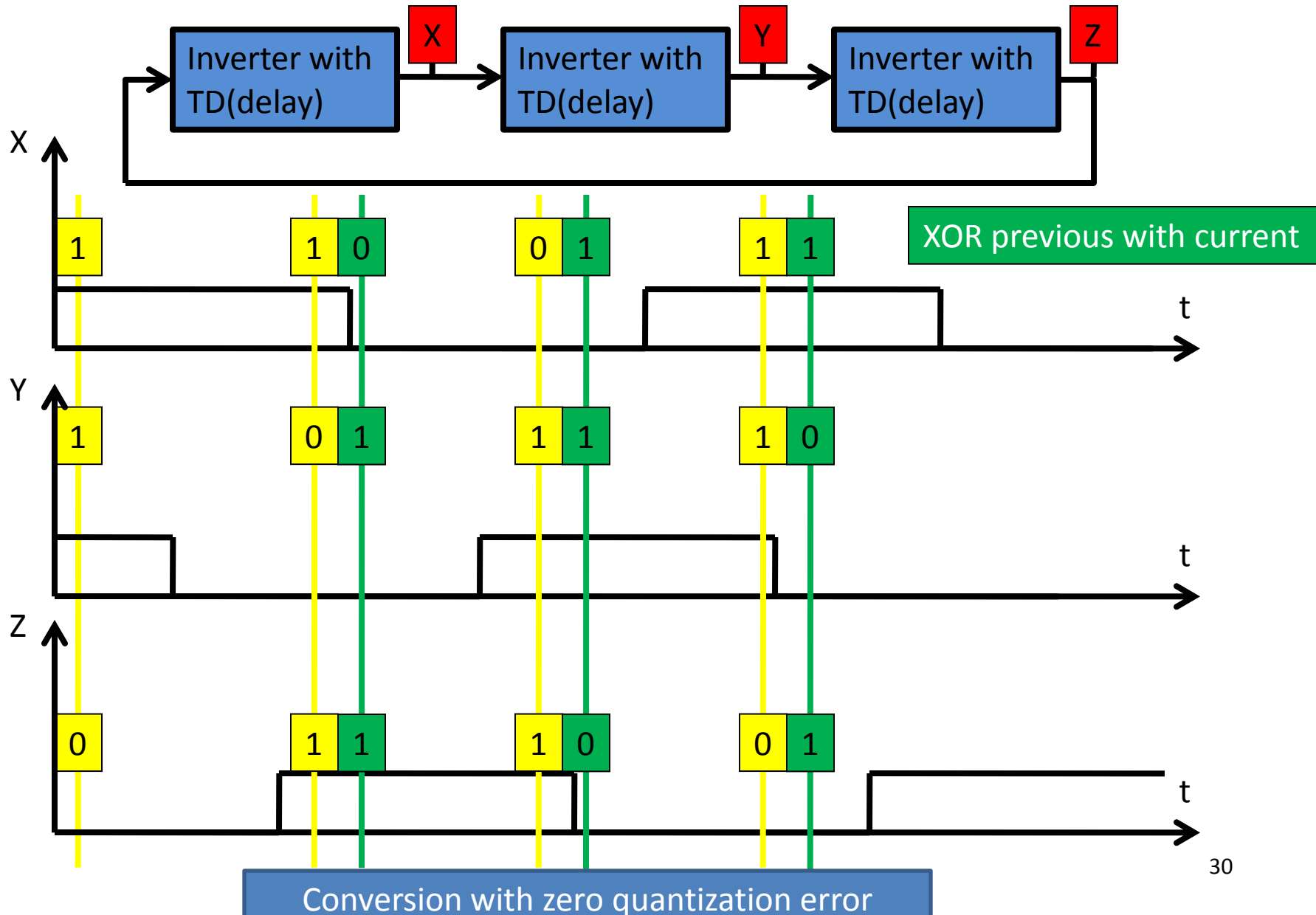
Ex2: Quantizing $V_{\text{tune}}=1/3V$ ($f_{\text{vco}}=f_{\text{s}}/3$)

Three sampling instances by period for $T_{\text{s}}=T_{\text{vco}}/3$

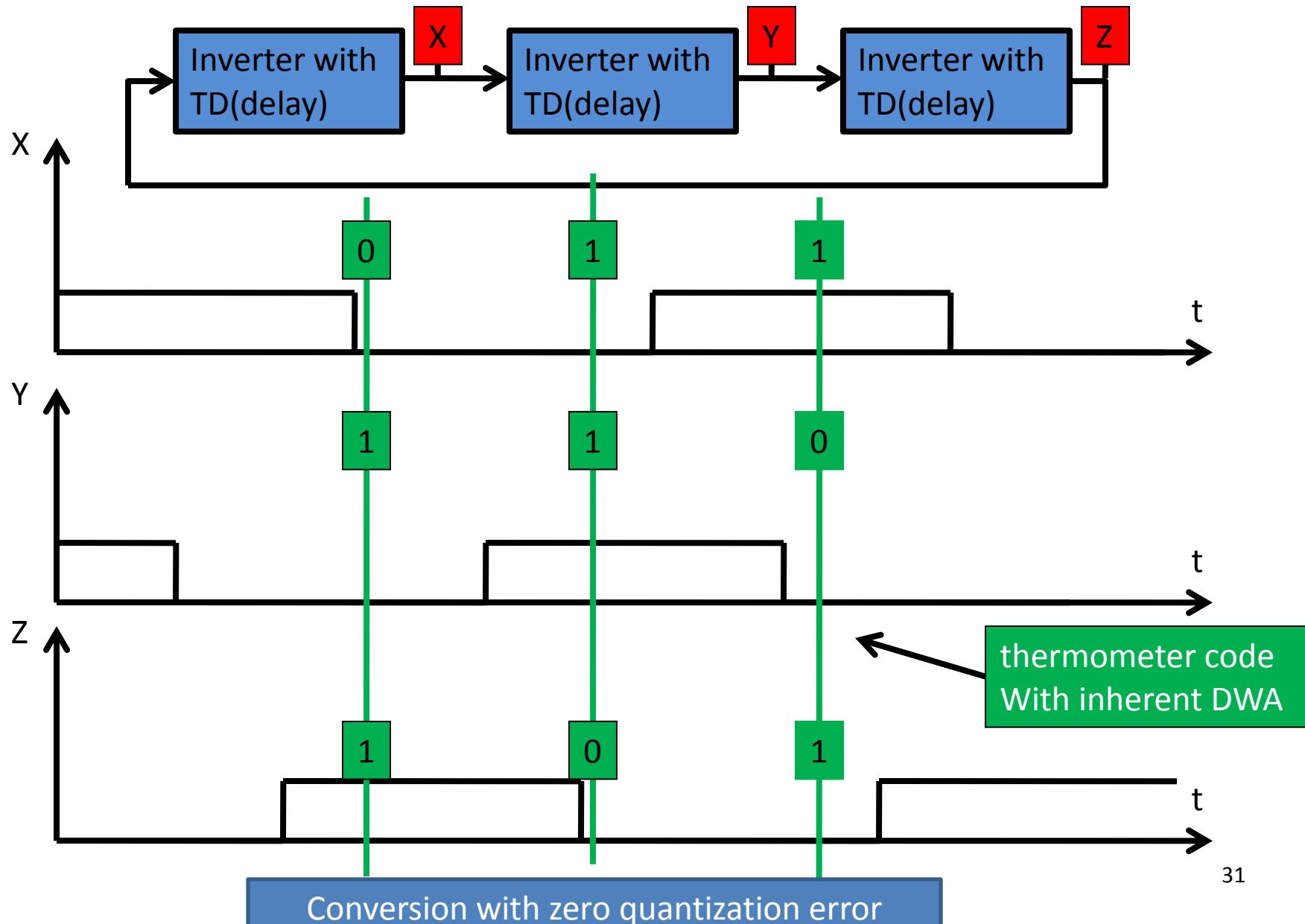


Conversion with zero quantization error

Ex2: Quantizing $V_{tune}=1/3V$ ($f_{vco}=f_s/3$)



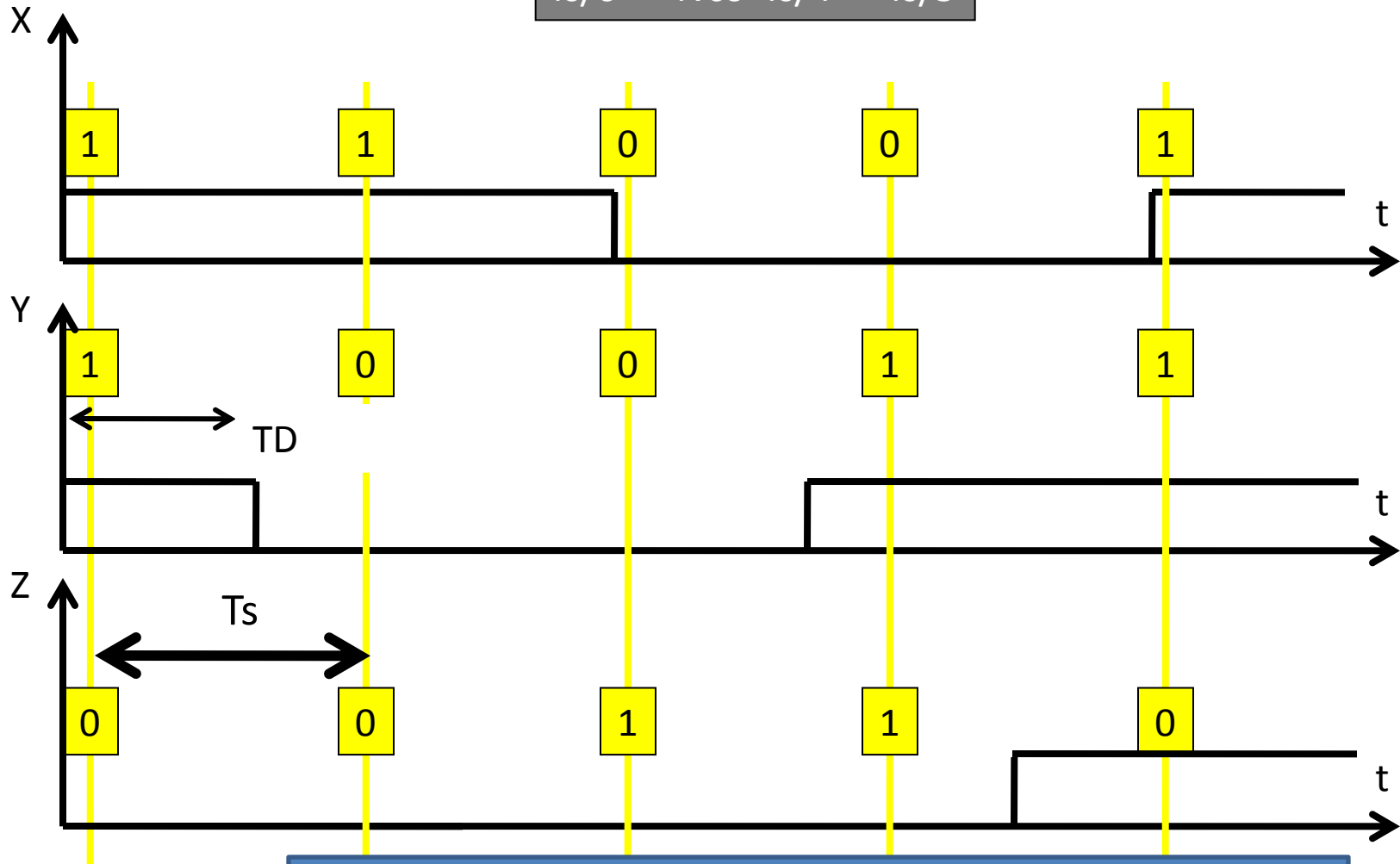
Ex2: Quantizing $V_{\text{tune}}=1/3V$ ($f_{\text{vco}}=f_{\text{s}}/3$)



Ex3: Quantizing $V_{\text{tune}}=1/4V$ ($f_{\text{vco}}=f_s/4$)

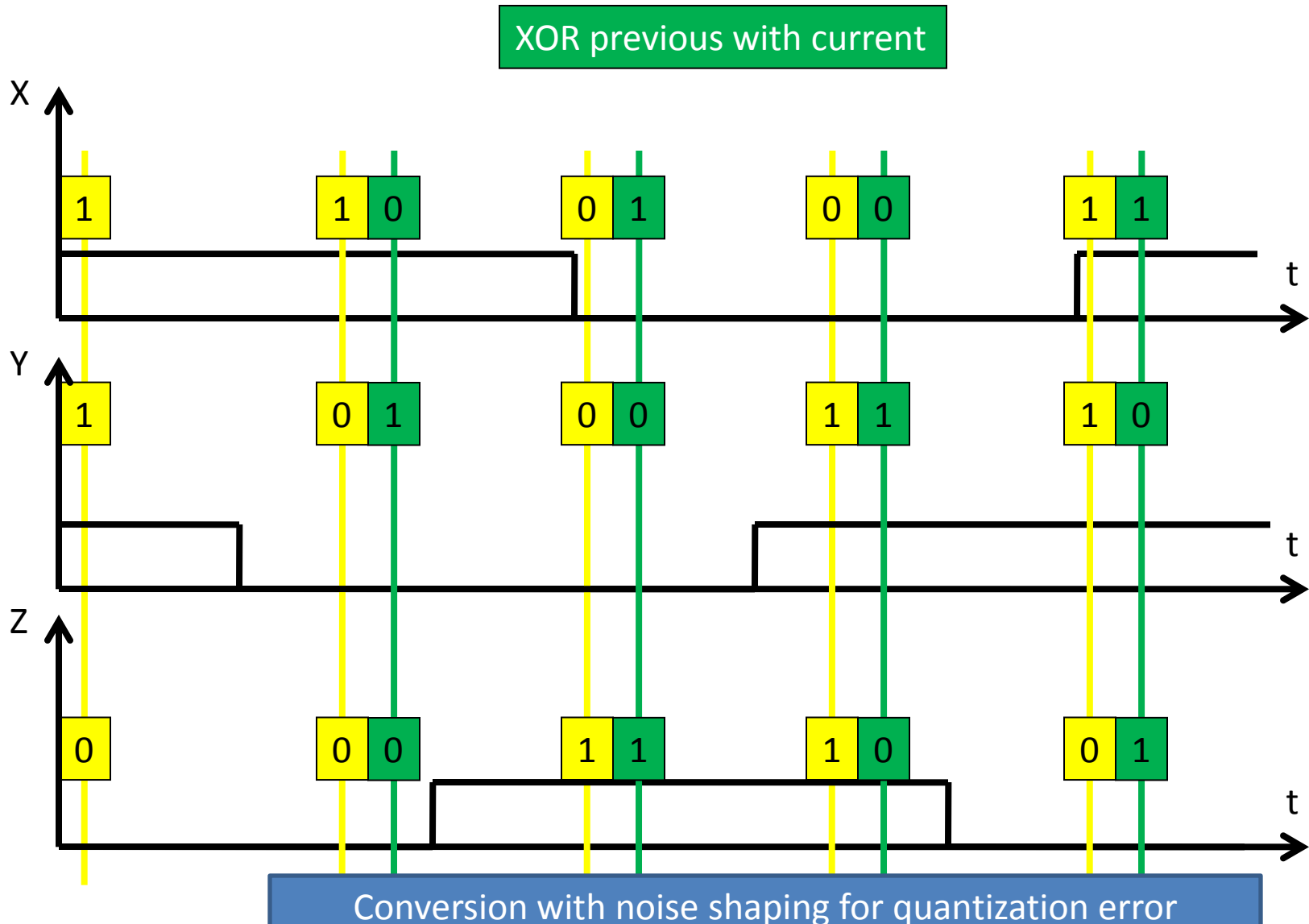
Four sampling instances by period for $T_s=T_{\text{vco}}/4$

$$f_s/6 < f_{\text{vco}}=f_s/4 < f_s/3$$



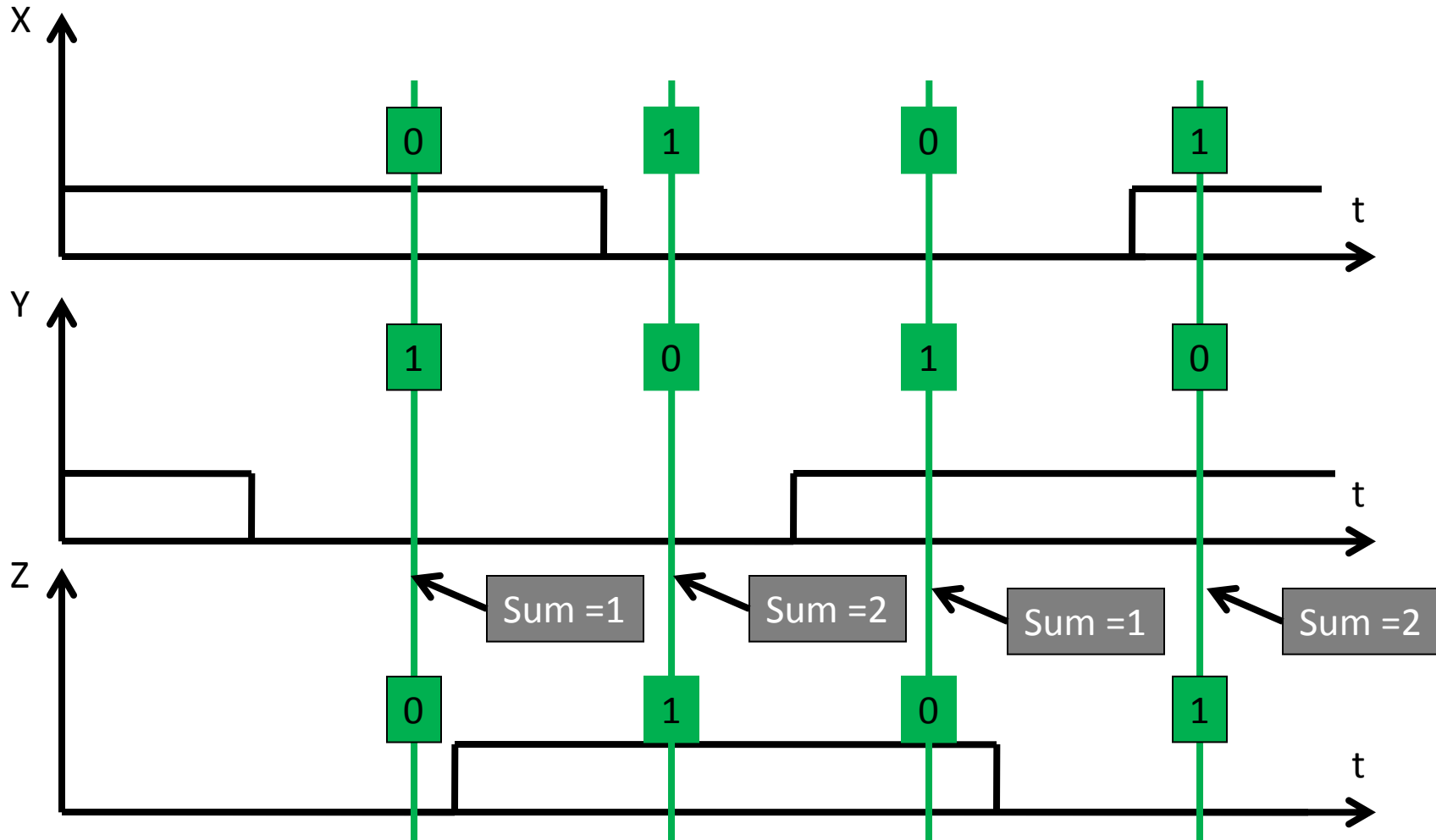
Conversion with noise shaping for quantization error

Ex3: Quantizing $V_{\text{tune}}=1/4V$ ($f_{\text{vco}}=f_s/4$)



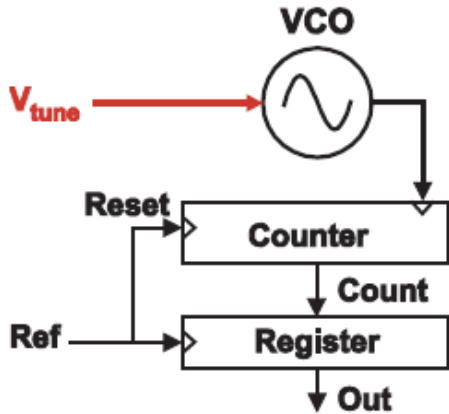
Ex3: Quantizing $V_{tune}=1/4V$ ($f_{vco}=f_s/4$)

Noise shaping: output toggles between 2 values for a constant input



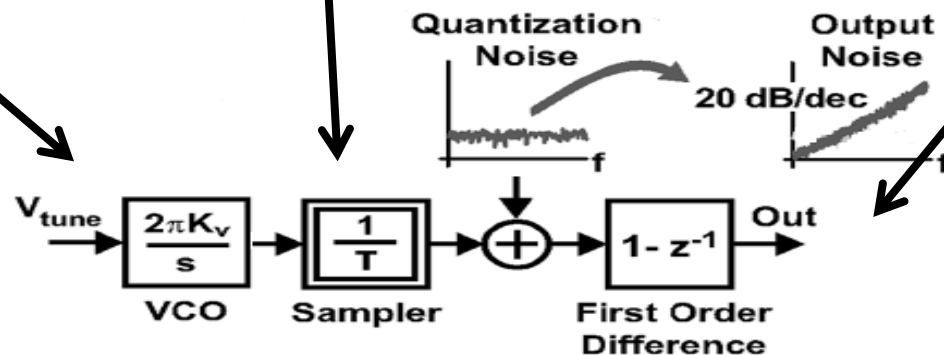
Conversion with noise shaping for quantization error

VCO-based quantizer Model



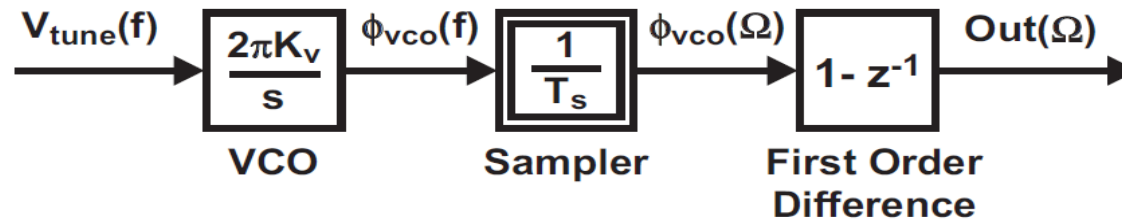
$$\Delta\Phi(t) = 2\pi \int (K_v v_{tune}(t)) dt$$

$$f_{VCO} = \frac{\Delta\Phi(t)}{\Delta t} = \frac{d}{dt} (2\pi \int (K_v v_{tune}(t)) dt)$$



VCO-based quantizer Model validity

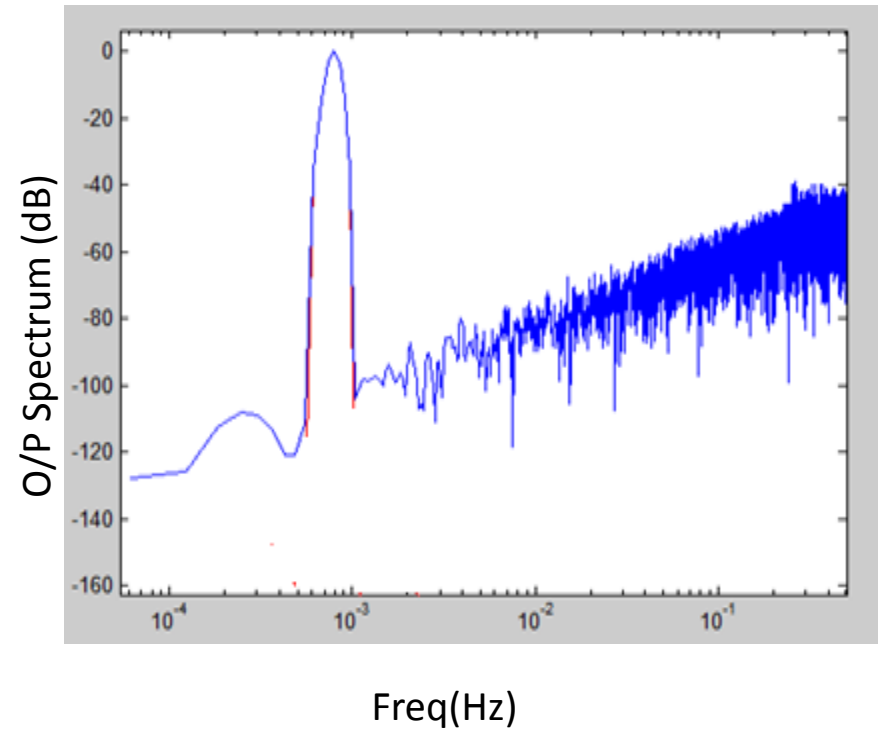
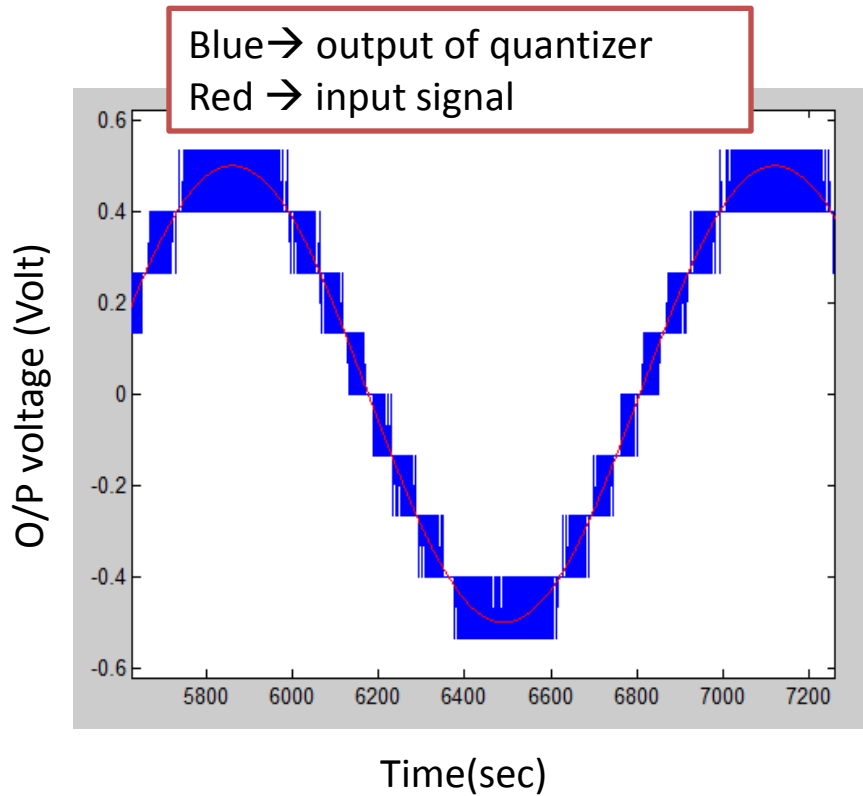
CT integrator and DT differentiator
Is it true that one is the inverse of the other ?



$$\begin{aligned}
 1 - z^{-1} &= 1 - e^{-sT_s} \\
 &= 1 - \left(1 + \frac{(-sT_s)^1}{1!} + \frac{(-sT_s)^2}{2!} + \frac{(-sT_s)^3}{3!} + \dots \right) \\
 &\approx sT_s \quad \longrightarrow \quad \omega \ll F_s.
 \end{aligned}$$

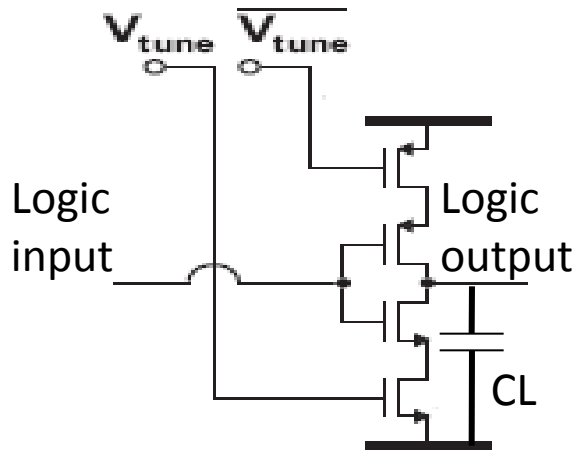
DT differentiation may be approximated as the inverse of the CT integration
Only for low frequencies with respect to F_s

VCO-based quantizer (4-bit) simulation



VCO-based quantizer Circuit drawbacks

VCO unit cell as two current sources

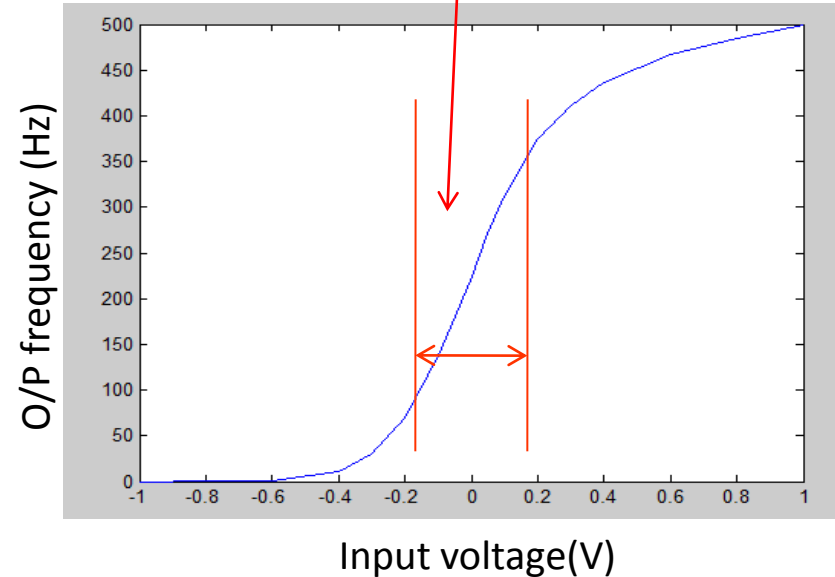


$$I_D = K(V_{gs} - V_t)^2$$

$$t_d = \frac{V_{swing} C_L}{I_D} \propto \frac{1}{V_{GS}^2}$$

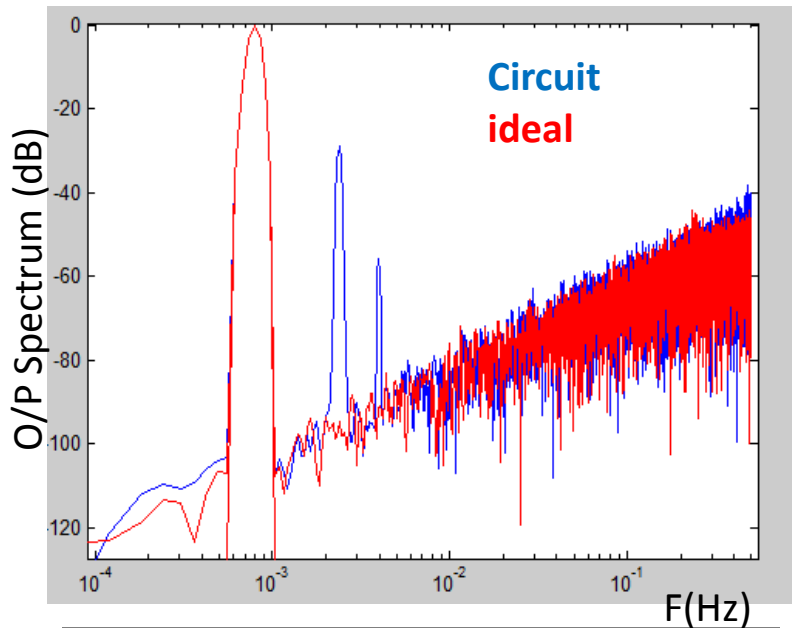
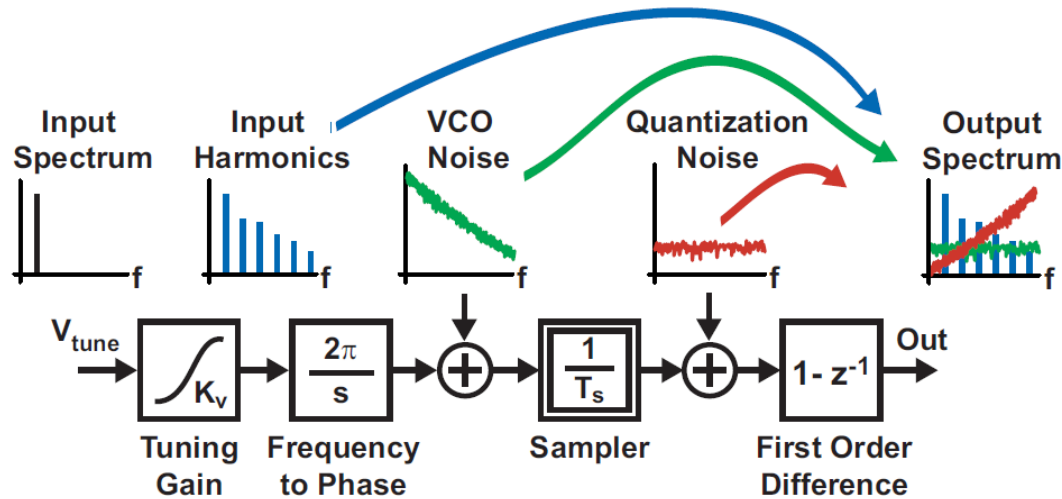
$$f_{osc} = \frac{1}{2Nt_d}$$

Small operation range

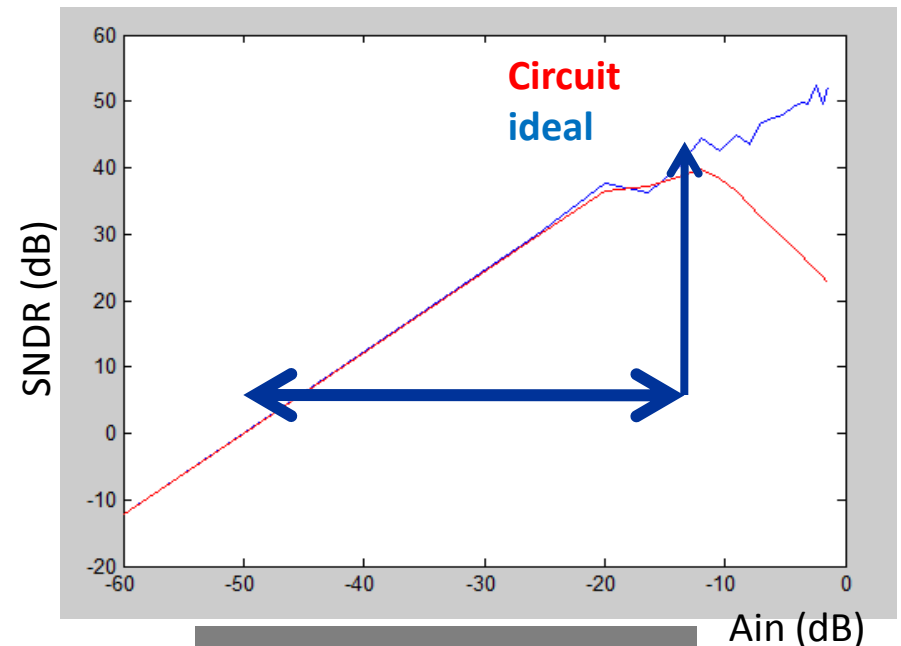


Non linear Relation and can be assumed linear on a small range

VCO-based quantizer Circuit drawbacks

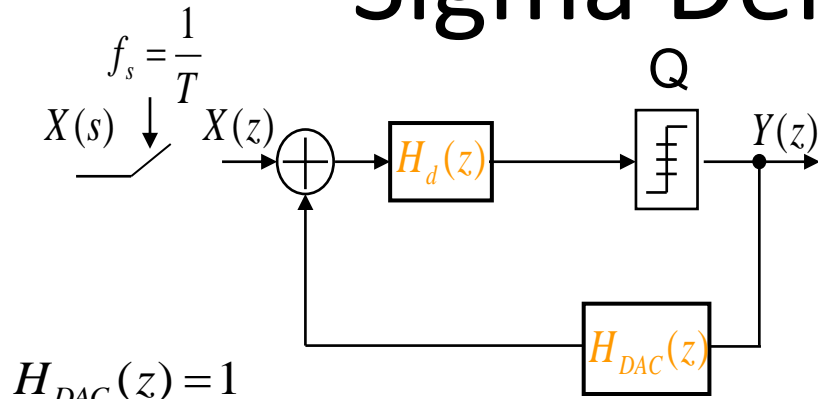


VCO-based quantizer spectrum



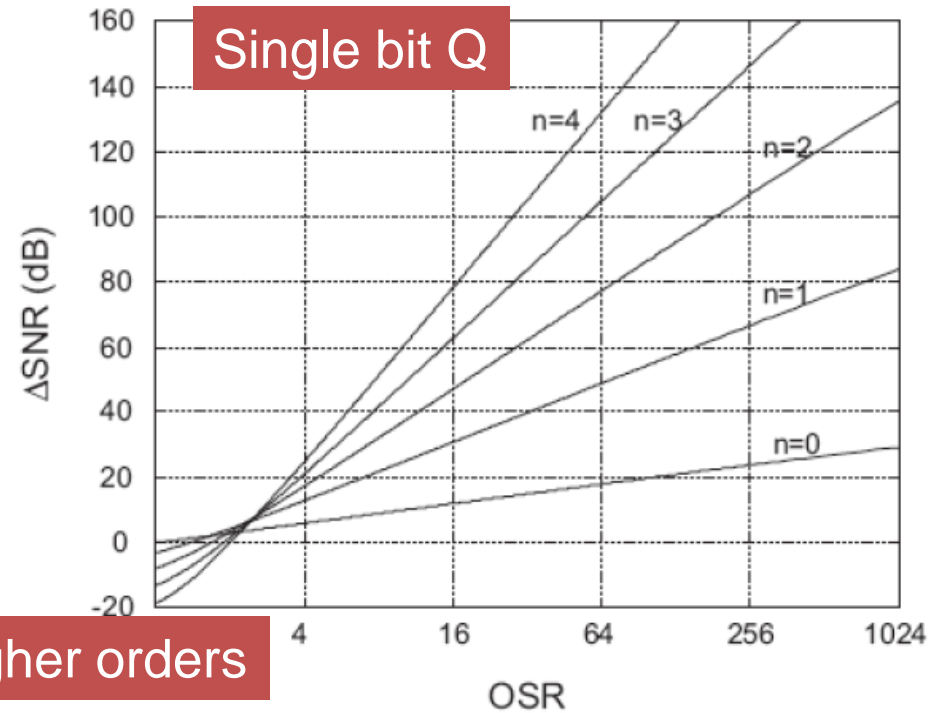
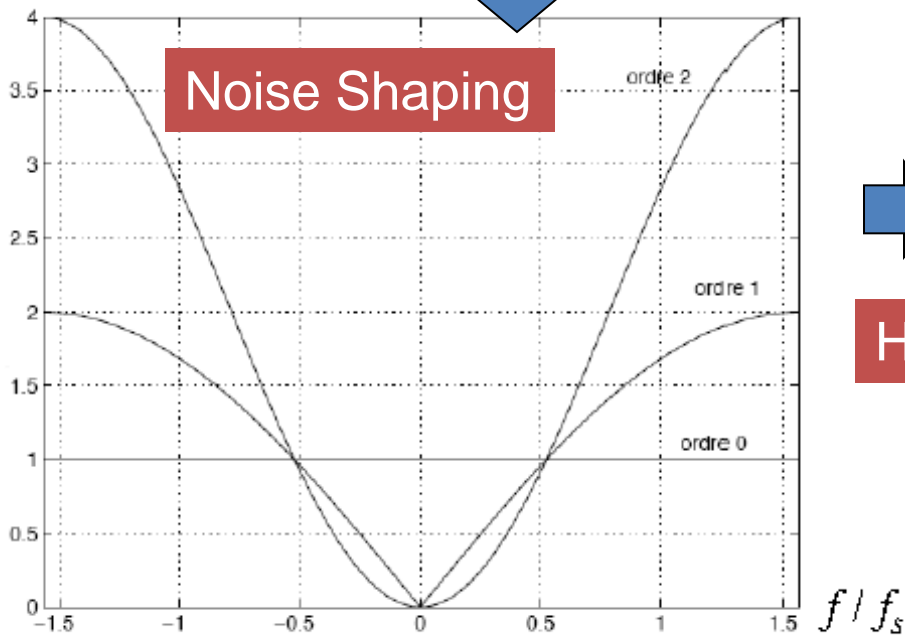
VCO quantizer(SNDR)

Sigma Delta ADC Review



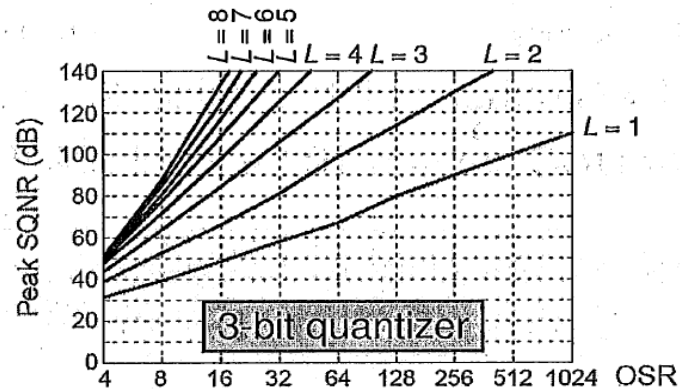
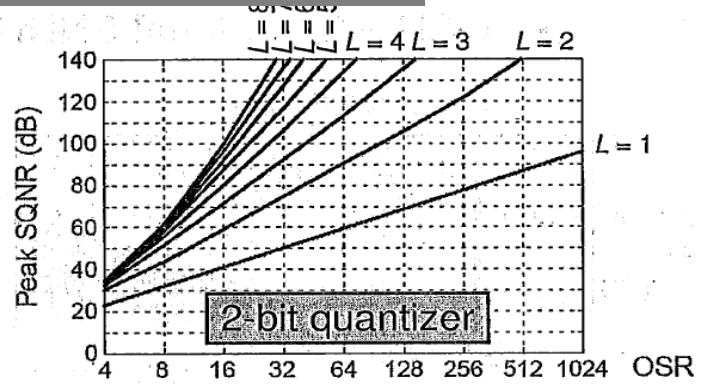
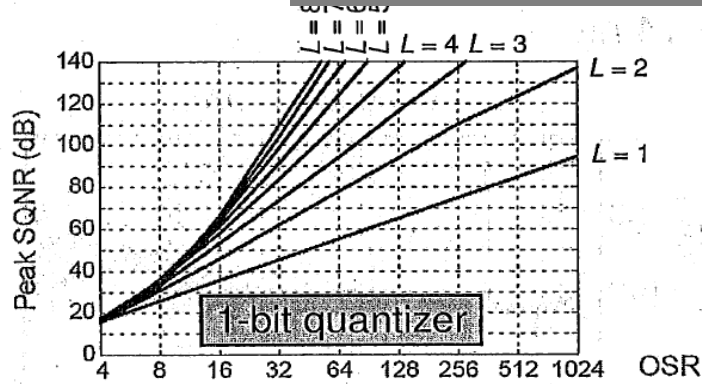
$$H_{DAC}(z) = 1$$

$$Y(z) = X(z) \frac{H_d(z)}{1 + H_d(z)} + Q(z) \frac{1}{1 + H_d(z)}$$



Multibit quantization

Higher SNR at lower OSR values



Estimated $SNR = \frac{3}{2} \left(\frac{2n+1}{\pi^{2n}} \right) (2^M - 1)^2 OSR^{2n+1}$

Order \swarrow \searrow Over Sampling ratio

Quantizer resolution \swarrow

Sigma Delta Design challenges

Wide BW

Low OSR

Resolution?

High Resolution

Multi-bit

Non-linearity?

High Filter Order

Stability?

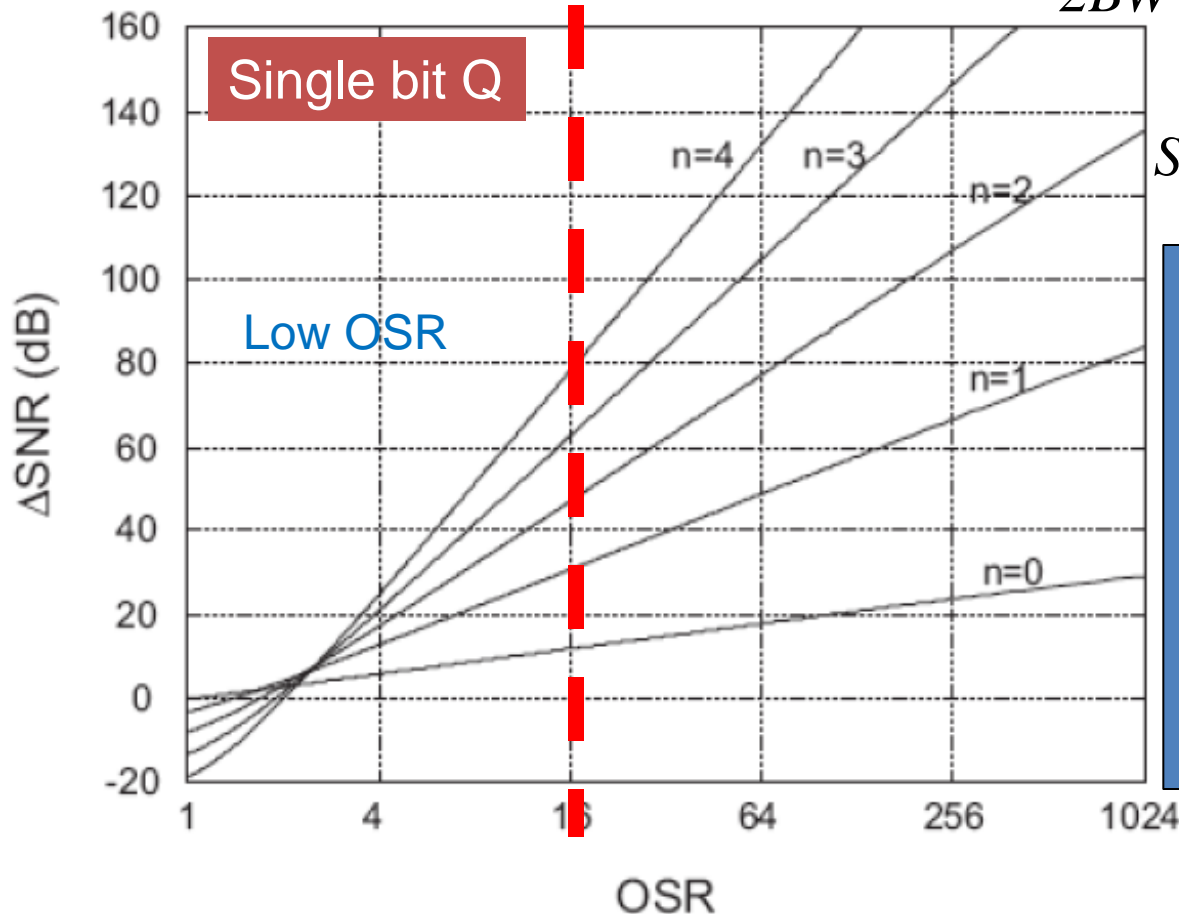
Low Power

Supply reduction

Analog blocks?

Design Challenges: Low OSR

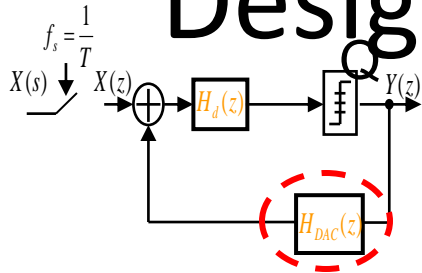
$$OSR = \left(\frac{f_s}{2BW} \right)$$



$$SNR = \frac{3}{2} \left(\frac{2n+1}{\pi^{2n}} \right) (2^M - 1)^2 OSR^{2n+1}$$

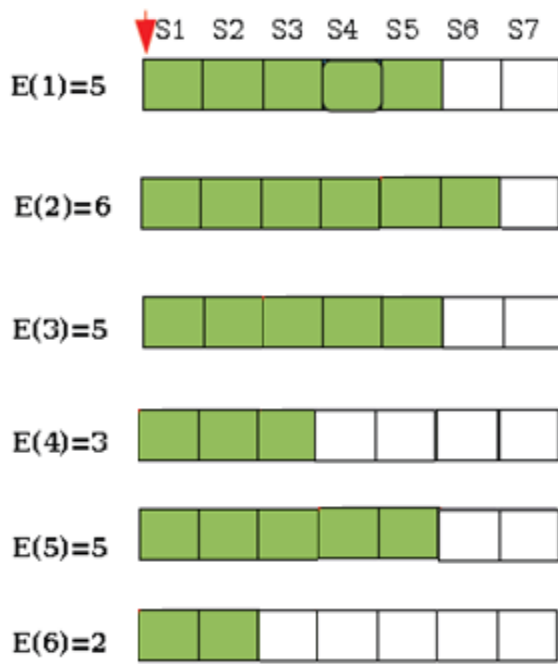
- Low OSR corresponds to lower SNR thus lower resolution
- Stabilized Single loop requires reducing NTF gain which will further decrease SNR

Design Challenge: Multibit DAC

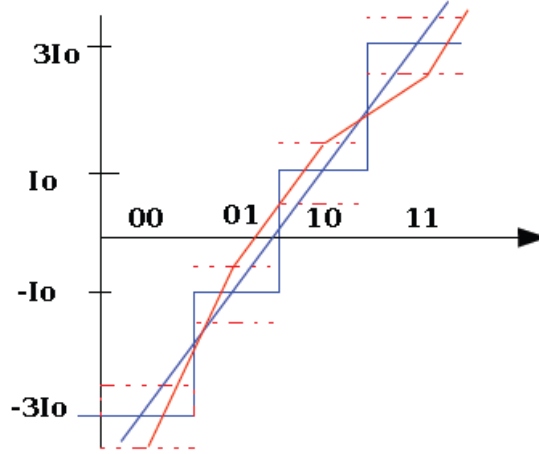


3bits DAC example

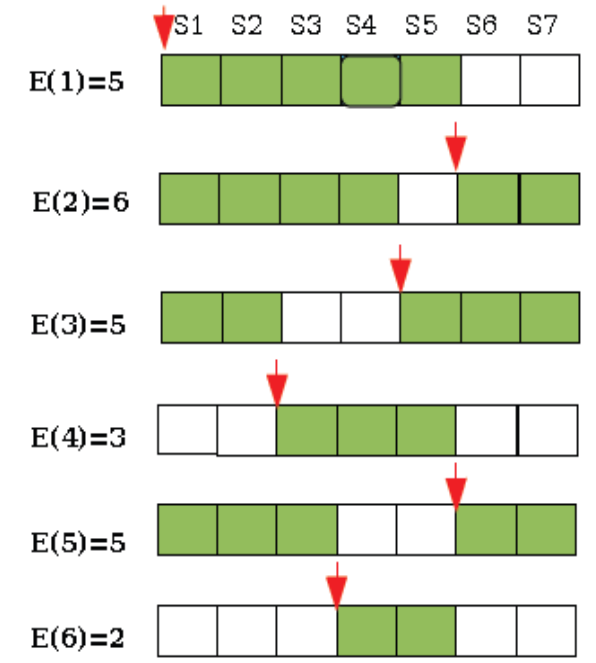
Thermometer code selecting DAC cells



Non linearity due to cell mismatch



DWA algorithm

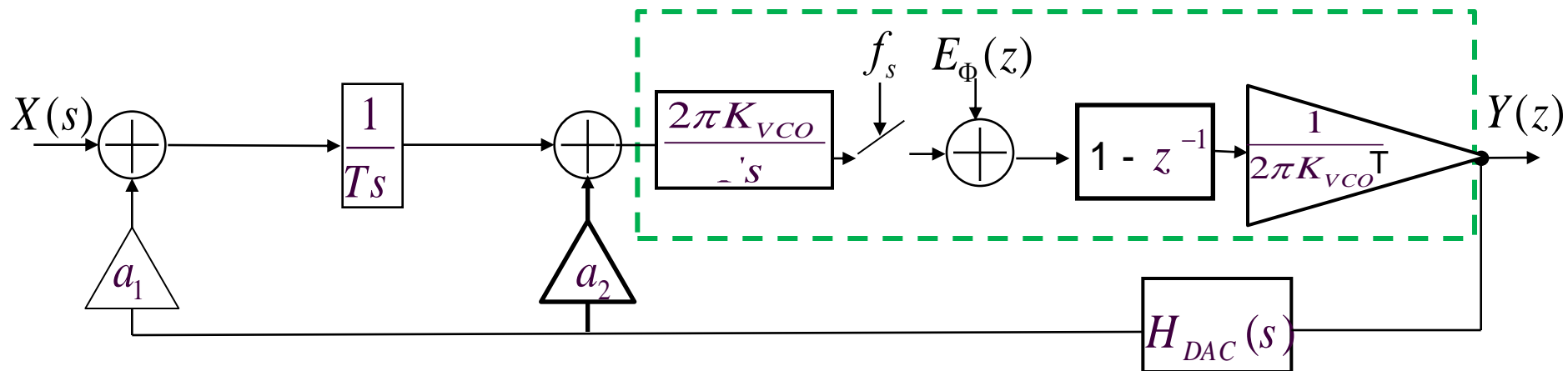


S -> Source de courant choisie
E -> Entrée du CNA

S -> Source de courant choisie
E -> Entrée du CNA

Second order SDM with VCO-based Quantizer

- Time Domain Quantization: Low Vdd (deep submicron) compatible
- No Comparators meta-stability, offsets and delay problems
- Inherent DWA without a dedicated circuit
- Saves an integrator (Mostly digital implementation)



SDM with VCO quantizer design(1)

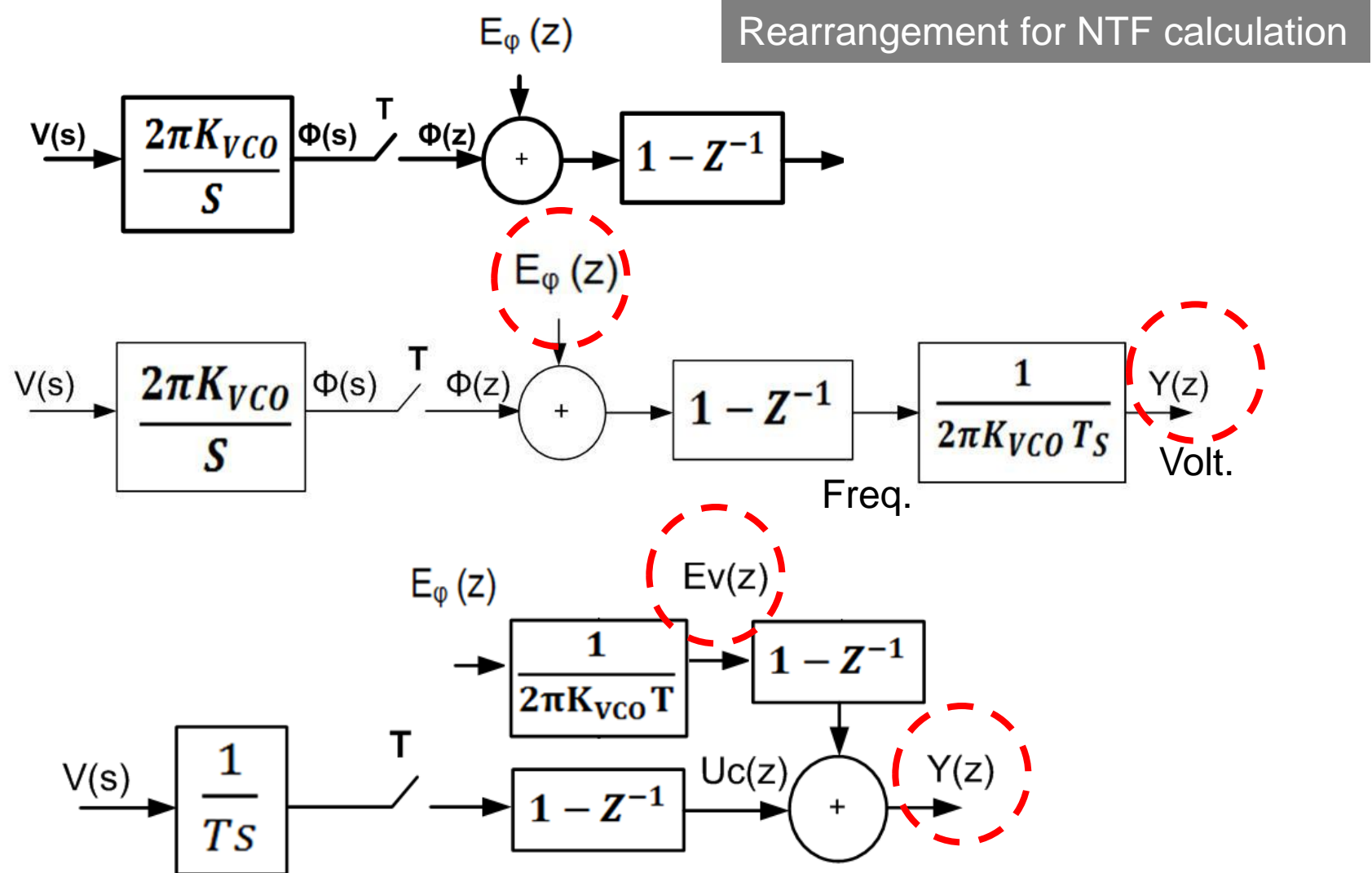
Exploit the 1st order noise shaping in the quantizer.

Transformation from Nth order DT to (N-1)th order CT with VCO quantizer

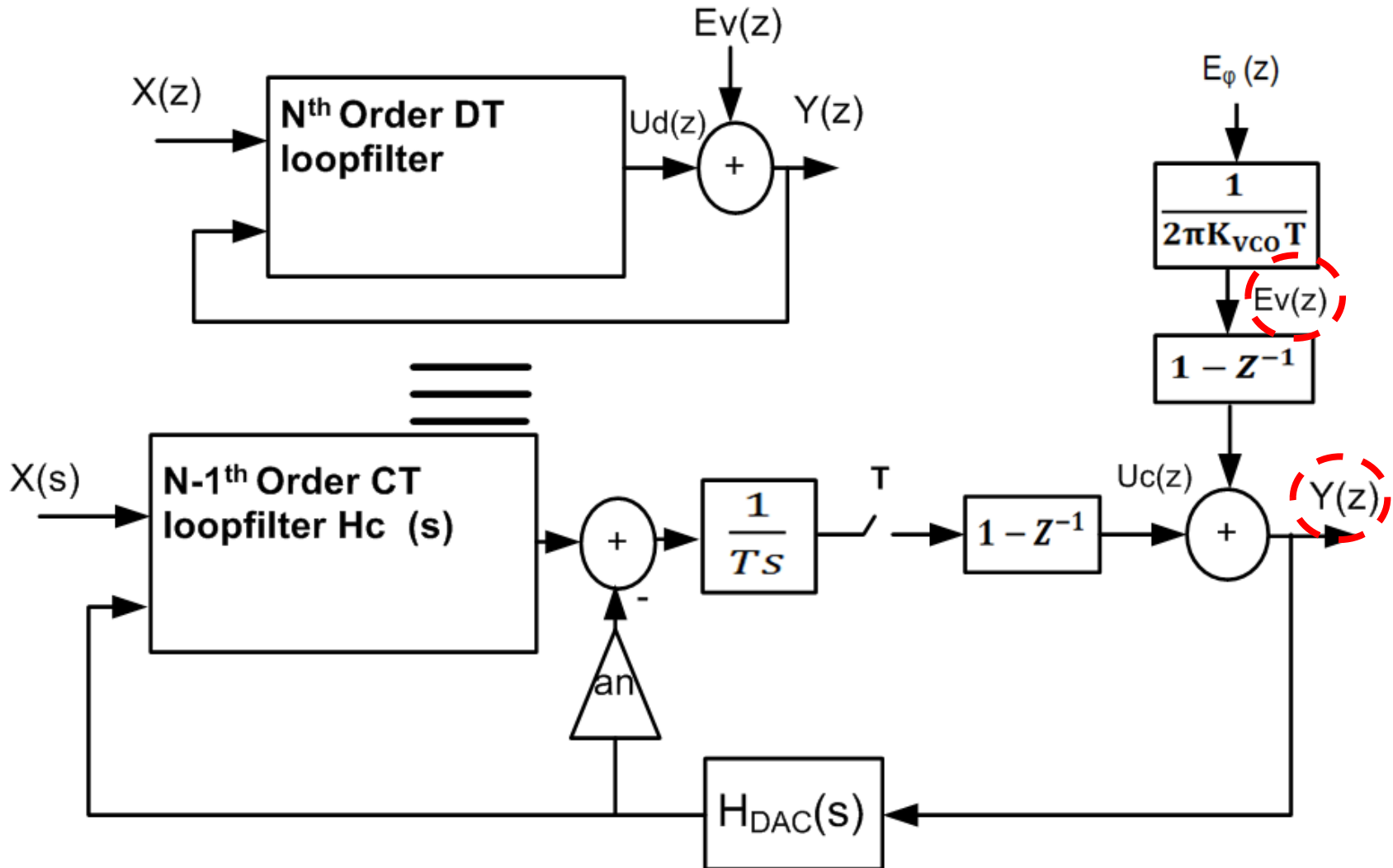
System Level Design

- 1) Get DT $\Delta\Sigma$ coefficients for Nth order Modulator (Schreier toolbox)
- 2) Get NTF_d(z) and G_d(z) of the DT modulator
- 3) Get NTF_c(z) and the corresponding loopgain G_c(z) of the CT modulator
- 4) Compare the two similar z orders in both NTF functions and to obtain the CT coefficients

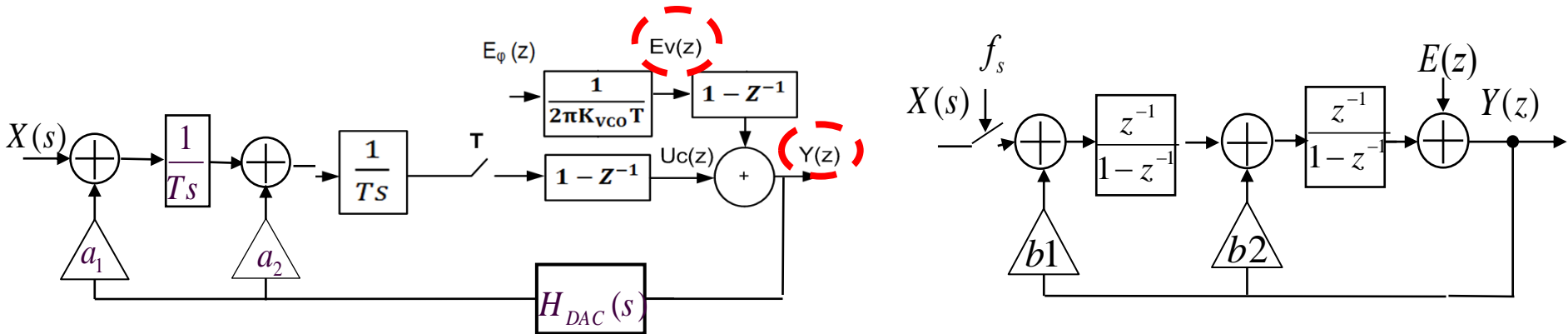
SDM with VCO quantizer design(2)



SDM with VCO quantizer design(3)



SDM with VCO quantizer design(4)

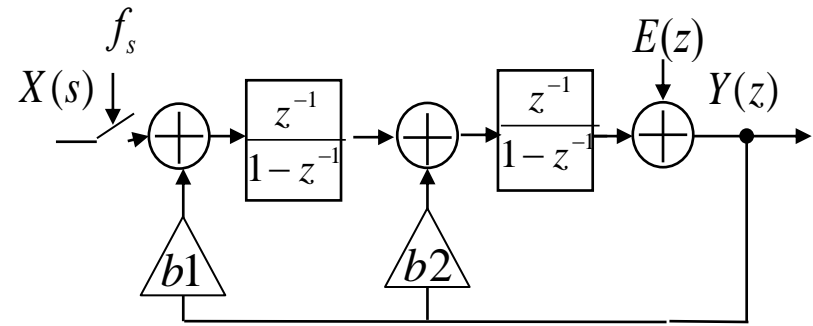
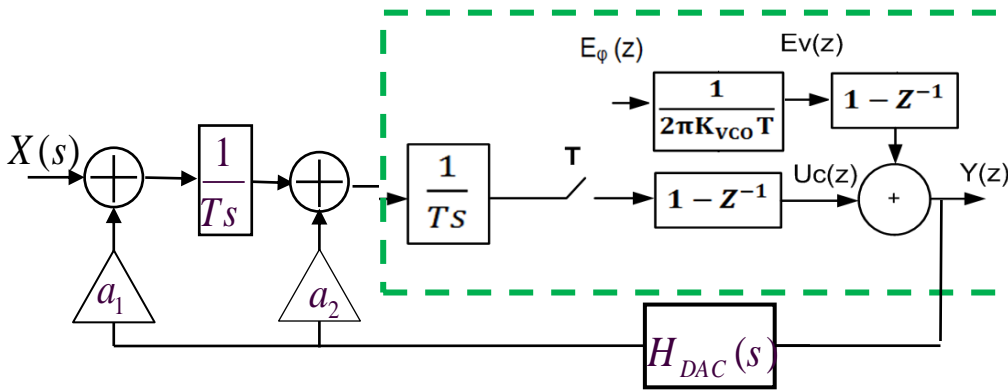


$$NTF_{VCO}(z) = \frac{Y(z)}{E_v(z)} = \frac{1 - z^{-1}}{1 - G_c(z)} = \frac{1}{1 - G'_c(z)}$$

$$G'_c(z) = \frac{-z^{-1}}{1 - z^{-1}} - Z \{H_{DAC}(s)H_C(s)\}$$

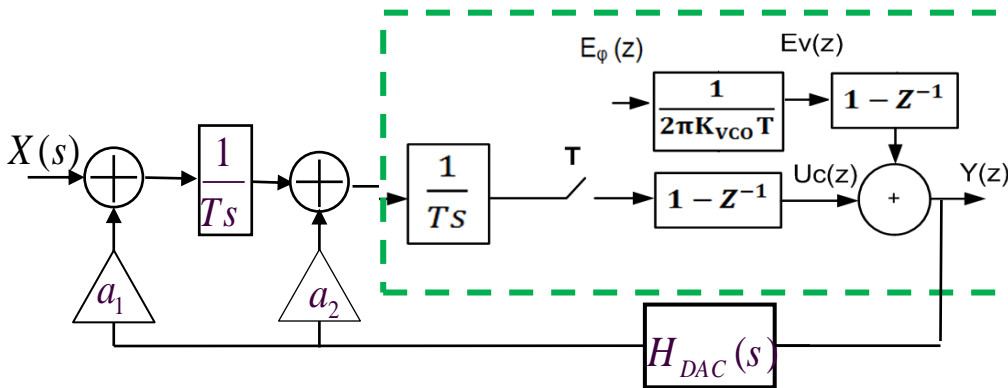
$$Z \left\{ H_{DAC}(s) \left(\frac{a_2}{Ts} + \frac{a_1}{T^2 s^2} \right) \right\} = Z \left\{ \left(\frac{1 - e^{-Ts}}{s} \right) \left(\frac{a_2}{Ts} + \frac{a_1}{T^2 s^2} \right) \right\}$$

SDM with VCO quantizer design(4)



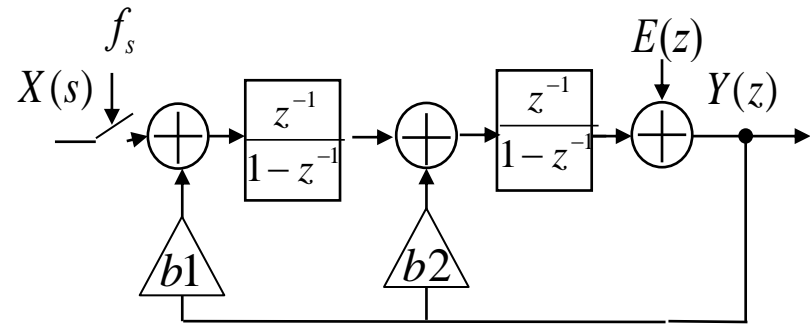
$$\begin{aligned}
 Z \left\{ H_{DAC}(s) \left(\frac{a_2}{Ts} + \frac{a_1}{T^2 s^2} \right) \right\} &= Z \left\{ \left(\frac{1 - e^{-Ts}}{s} \right) \left(\frac{a_2}{Ts} + \frac{a_1}{T^2 s^2} \right) \right\} \\
 &= (1 - z^{-1}) * Z \left\{ \left(\frac{a_2}{Ts^2} + \frac{a_1}{T^2 s^3} \right) \right\} \\
 &= (1 - z^{-1}) * \left(a_2 \frac{-z}{(z-1)^2} - \frac{a_1}{2} \frac{z(z+1)}{(z-1)^3} \right) \\
 &= (z-1) * \left(a_2 \frac{-1}{(z-1)^2} - \frac{a_1}{2} \frac{(z+1)}{(z-1)^3} \right) \\
 &= \left(a_2 \frac{-1}{(z-1)} - \frac{a_1}{2} \frac{(z+1)}{(z-1)^2} \right) \\
 &= \frac{(-a_2 - a_1/2 - 1)z + (a_2 - a_1/2 + 1)}{(z-1)^2}
 \end{aligned}$$

SDM with VCO quantizer design(4)



$$NTF_{VCO}(z) = \frac{Y(z)}{E_V(z)} = \frac{1-z^{-1}}{1-G_c(z)} = \frac{1}{1-G'_c(z)}$$

$$G'_c(z) = \frac{(-a_2 - a_1/2 - 1)z + (a_2 - a_1/2 + 1)}{(z-1)^2}$$



$$NTF_D(z) = \frac{Y(z)}{E(z)} = \frac{1}{1-G_d(z)}$$

$$G_d(z) = \frac{-(b_2)z + b_2 - b_1}{(z-1)^2}$$

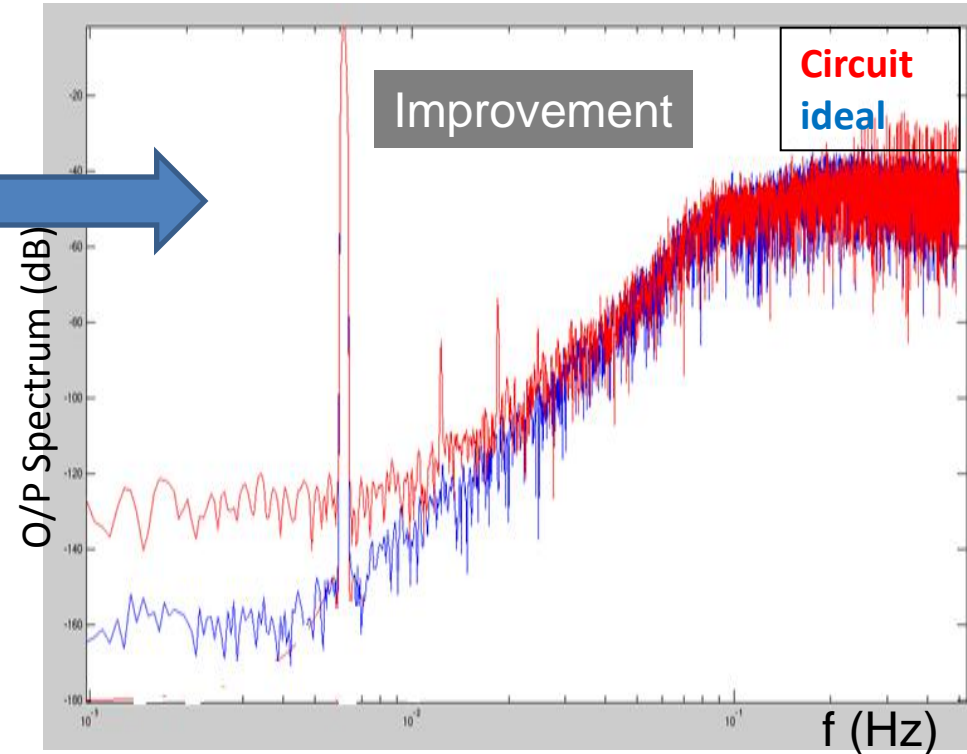
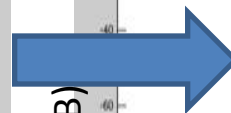
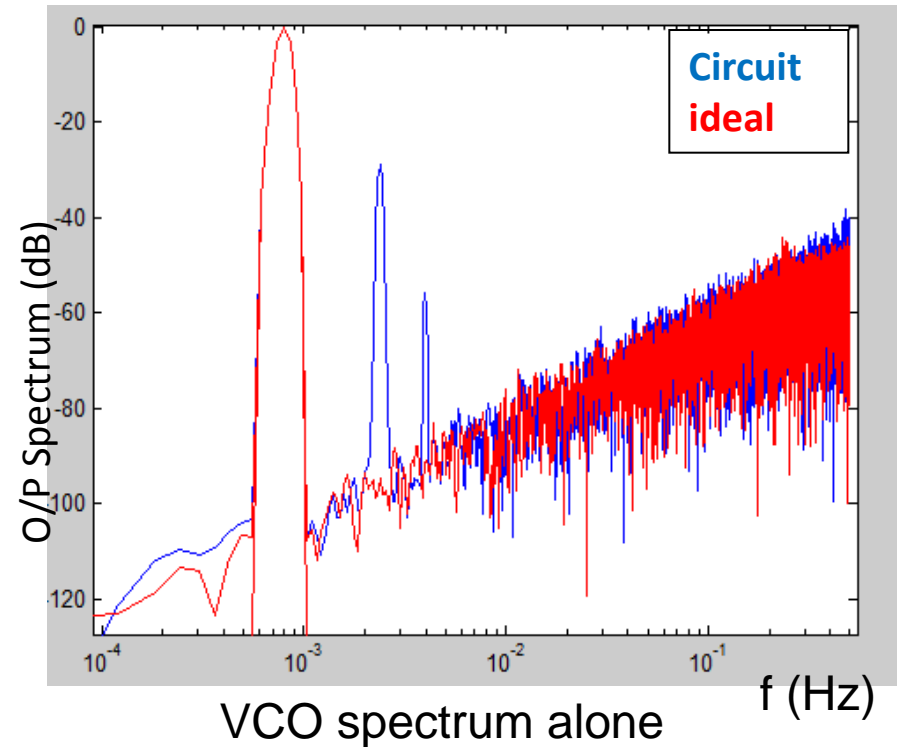
$$-b_2 = -a_0 - \frac{a_1}{2} - 1$$

$$b_2 - b_1 = a_2 - \frac{a_1}{2} + 1$$

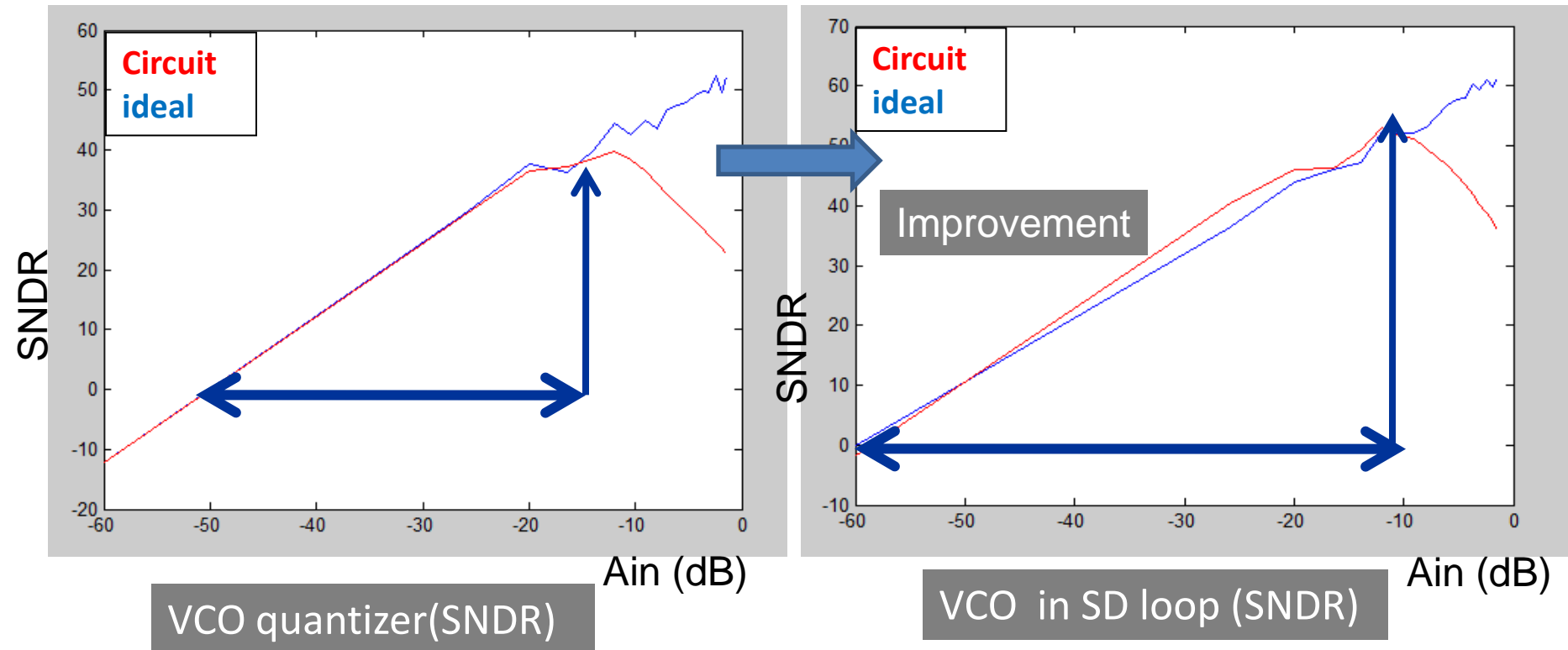
$$b_1 = a_1$$

$$b_2 - \frac{b_1}{2} - 1 = a_2$$

Non ideal VCO-based quantizer in SD

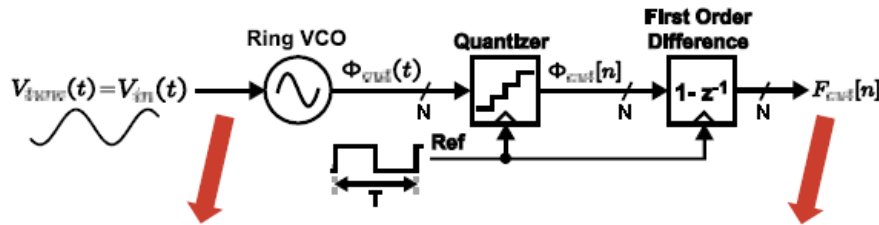


Non ideal VCO-based quantizer in SD

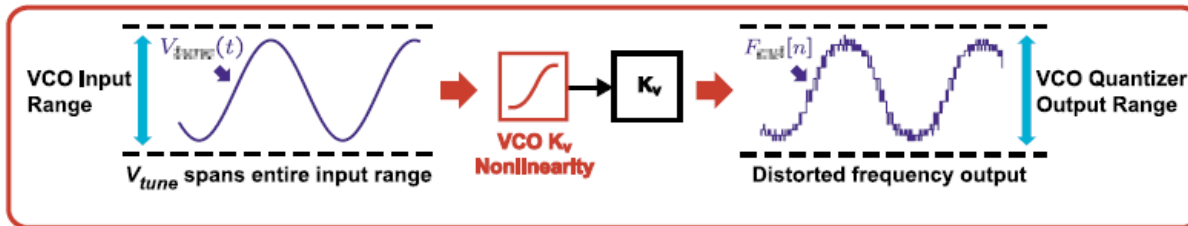


Linearization through feedback

Prior Architectures: Voltage-to-Frequency Quantization

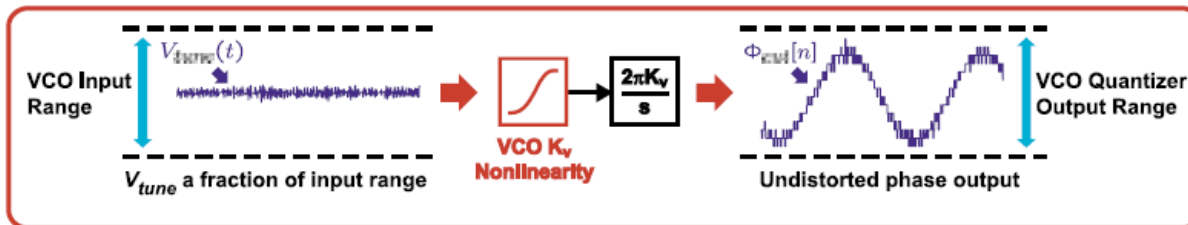
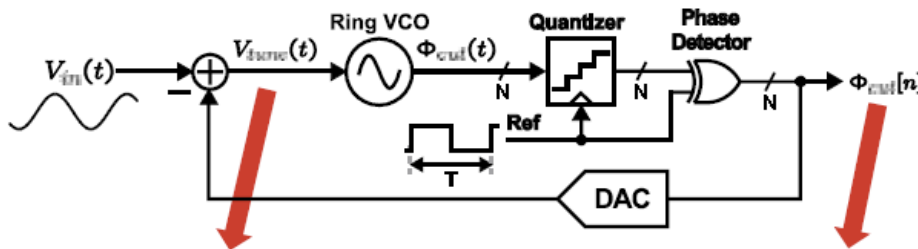


Using Negative feedback
This will make the VCO always running very close to its free running frequency

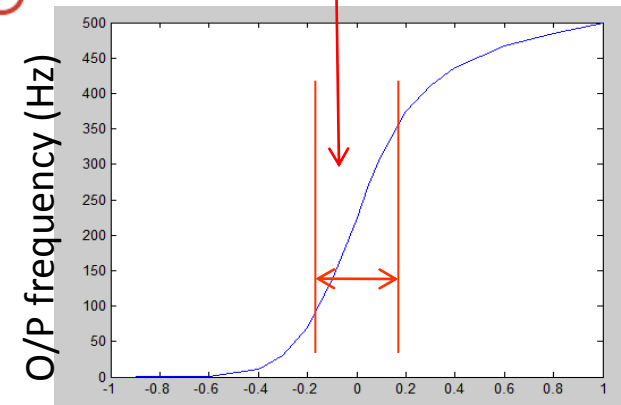


(a)

Proposed Architecture: Voltage-to-Phase Quantization



Small operation range



Input voltage(V)

Summary & Conclusion

- TDC are potential candidates for modern wide band ADC
- They achieve good resolution and BW efficiently (low power)
- They are compatible with technology scaling thus allowing further efficiency in terms of power and area
- Using TDC techniques in ADC need analog to time conversion which is usually non-linear.
- VCO-based quantizer has an important noise shaping property
- The use of VCO-based quantizer as a multi-bit quantizer in SDM decreases the non-linearity due to the SDM loop filter gain.
- Further suppression of non-linearities is possible using Negative Feedback

References

- M. Z. Straayer, M. H. Perrott, "A 12-Bit, 10-MHz Bandwidth, Continuous-Time ADC With a 5-Bit, 950-MS/s VCO-Based Quantizer," *IEEE JSSC*, vol. 43, NO. 4, pp.805-814, April 2008
- M. Z. Straayer, "Noise Shaping Techniques for Analog and Time to Digital Converters Using Voltage Controlled Oscillators", *PhD thesis, MIT*, June.2008
- H. Aboushady and M.-M. Louerat, "Systematic approach for discrete-time to continuous-time transformation of modulators," in *Proc. IEEE International Symposium on Circuits and Systems, (ISCAS'02)*, vol. 4, 2002, pp. IV-229-IV-232
- H. Aboushady, "Conception En Vue De La Realization De Convertisseurs Analogique- Numerique $\Sigma\Delta$ Temps-Continu Mode courant", *Phd thesis, UPMC*, Lip6,Jan.2002.
- M. J. Park, "A 4th Order Continuous-Time $\Delta\Sigma$ ADC with VCO-Based Integrator and Quantizer", *PhD thesis, MIT*, Feb.2009
- T. E. Rahkone, J. T. Kostamovaara, "The Use of Stabilized CMOS Delay Lines for the Digitization of Short Time Intervals", *IEEE JSSC*, vol. 28, no. 8, Aug. 1993
- J.Kim, S. Cho, "A time-based analog-to-digital converter using a multi-phase voltage controlled oscillator," *ISCAS 2006*.