Systematic Design Method for LC Bandpass $\Sigma\Delta$ Modulators with Feedback FIRDACs

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***Abstract*— In this paper, a generalized technique for the design automation of $\frac{f_s}{4}$ LC bandpass $\Sigma\Delta$ modulators using feedback FIRDACs is proposed. The FIRDACs are used to increase the degrees of freedom in order to perform an exact equivalence with high order discrete-time $\Sigma\Delta$ modulators and also to allow a more efficient circuit implementation of the LC filter. The design technique is based on Discrete Time-Continuous Time equivalence simplified by using the method of partial fractions expansion. The excess loop delay is taken into account without making more difficult the calculations since we define how to get the orders of the FIRDACs. Several examples of design are simulated with different values of excess loop delay.**

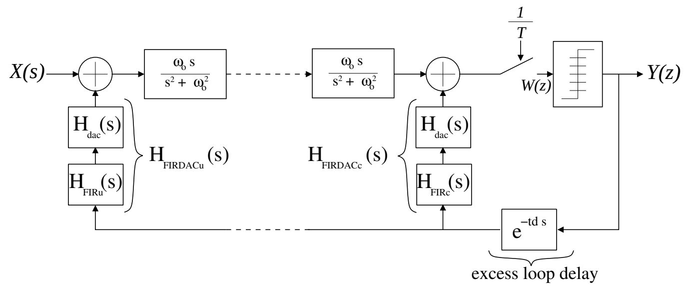

Fig. 1. Model of an LC $\Sigma\Delta$ modulator without intermediate nodes

I. Introduction

Recent years have shown an increasing interest to digitize the input signal near to the front end of the antenna so as to push more signal processing functions into the digital domain [1]. LC filter based $\Sigma\Delta$ modulators have been considered for direct digitization at RF frequencies [2], [3] since they are more suitable for high frequency applications. The use of LC filters implies a limited degrees of freedom for the design of the Noise-Transfer-Function (NTF) [4], [5]. In [6] a method based on FIRDACs has been presented to solve this problem. It has been shown that by using FIRDACs, internal nodes could be removed, thus allowing a more efficient circuit implementation (Fig.1).

The coefficients of the FIRDACs are computed by using the Discrete Time-Continuous Time (DT-CT) transformation technique, which consists in equating, in the Z domain, the loop-gains of the discrete-time (DT) and continuous-time (CT) modulators [4], [7], respectively $G_d(z)$ and $G_c(z)$:

$$G_d(z) \equiv G_c(z) = \frac{W(z)}{Y(z)} \tag{1}$$

To alleviate the performance degradation due to excess loop delay [8], we take it into account in the CT loop gain (Fig. 1) and thus in the calculation of the CT coefficients [9]. Nevertheless, this leads to make the DT-CT equivalence more difficult to achieve as the order of the FIRDACs may vary with the value of the excess loop delay. Therefore, the automation of the CT coefficients calculation becomes painstaking since we have to determine for each given architecture and each value of the excess loop delay the suitable changes that should occur to achieve the DT-CT equivalence.

In this paper, we propose a systematic technique based on partial fractions expansion to calculate the FIRDACs coefficients in order to make the NTF of an $\frac{f_s}{4}$ LC $\Sigma\Delta$ modulator identical to a DT Bandpass (BP) $\Sigma\Delta$. In section II, general expressions for the loop gains of the CT $\Sigma\Delta$ modulators considered are presented. Starting from these expressions, a systematic technique to compute the orders and the coefficients of the FIRDACs is proposed in section III. Finally, this technique has been applied to different LC $\Sigma\Delta$ modulator orders with different values of excess loop delay, section IV.

II. LC $\Sigma\Delta$ Modulator using 2 FIRDACs

The considered architecture of LC $\Sigma\Delta$ modulator depicted Fig.1 using only 2 FIRDACs, the loop gain can be expressed by considering separately both FIRDACs :

$$\begin{aligned} Gc(z) \quad &= \quad \underbrace{\mathcal{Z}\left\{\left(\frac{\omega_o s}{s^2+\omega_o^2}\right)^{\frac{n}{2}} H_{dac}(s) H_{FIR_U}(s) e^{-t_d s}\right\}}_{H_U(z)} \\ &+ \quad \underbrace{\mathcal{Z}\left\{H_{dac}(s) H_{FIR_C}(s) e^{-t_d s}\right\}}_{H_C(z)} \end{aligned} \tag{2}$$

where $H_U(z)$ and $H_C(z)$ are respectively called useful transfer function and compensation transfer function, n is the order of the LC filter and ω_o determines the center frequency of the LC filter ($\omega_o = \frac{2\pi}{4T}$).

The FIRDACs are composed of digital-to-analog converters (DAC) with gains (u_i and c_i) which are separated by half-cycle delays. The choice of using half-cycle delays instead of full-cycle delays for FIR filters is to have more degrees of freedom. In the following we shall consider separately the transfer function of the DAC, $H_{dac}(s)$, which has been factorized in the FIRDAC expression, and the transfer functions of the FIRs, $H_{FIR}(s)$, with the coefficients and delays. The expressions of

$H_{FIR}(s)$ are the following :

$$\begin{cases} H_{FIR_U}(s) = \sum_{i=0}^{i=M_u-1} u_i e^{-\frac{isT}{2}} \\ H_{FIR_C}(s) = \sum_{i=0}^{i=M_c-1} c_i e^{-\frac{isT}{2}} \end{cases} \quad (3)$$

where M_u and M_c are respectively the order of the useful FIRDAC and the order of the compensation FIRDAC, and u_i and c_i their respective coefficients.

The DAC considered gives a sine-shaped feedback signal (Fig.2) which is less sensitive to the clock-jitter [10] and thus is more adapted to an analog-to-digital conversion at RF frequencies. Its transfer function is written as:

$$H_{dac}(s) = \frac{\omega_{dac}^2(1-e^{-sT})}{s(s^2+\omega_{dac}^2)} \quad (4)$$

where ω_{dac} determines the frequency of the DAC output signal.

By simplifying and considering the properties of the modified Z-transform used due to the delays less than a sampling period, the useful transfer function from equation (2) can be written as:

$$H_U(z) = \frac{z-1}{z} \sum_{i=0}^{M_u-1} z^{-n_i} u_i \mathcal{Z}_{m_i} \left\{ \underbrace{\frac{\omega_{dac}^2\ \omega_o^{\frac{n}{2}}\ s^{\frac{n}{2}-1}}{(s^2+\omega_{dac}^2)(s^2+\omega_0^2)^{\frac{n}{2}}}}_{A(s)} \right\} \quad (5)$$

where,

- $n_i = \lfloor \frac{i}{2} + \frac{t_d}{T} \rfloor$ where $\lfloor x \rfloor$ denotes the greatest integer inferior or equal to x.
- $m_i = 1 - (\frac{i}{2} + \frac{t_d}{T}) + \lfloor \frac{i}{2} + \frac{t_d}{T} \rfloor$

Since i is an integer, according to the above expression we can say that m_i has only 2 different values. We deduce that the number of modified Z-transforms to compute is always 2.

In the same manner, for the compensation transfer function from equation (2), we define the following expression:

$$H_C(z) = \frac{z-1}{z} \sum_{i=0}^{M_c-1} z^{-\ell_i} c_i \mathcal{Z}_{m_i} \left\{ \underbrace{\frac{\omega_{dac}^2}{s(s^2+\omega_{dac}^2)}}_{B(s)} \right\} \quad (6)$$

where,

- $\ell_i = \lfloor \frac{i}{2} + \frac{t_d}{T} \rfloor$
- m_i are the same as the useful transfer function.

In the following we will compute the modified Z-transform of the expressions (5) and (6) and find the values of the coefficients u_i and c_i to achieve the equivalence with the DT loop gain.

III. Design Method:Partial Fractions Identification

The partial fractions expansion of a proper fraction[1] is unique. Therefore, we can compute the coefficients of the FIR-DACs by identifying the partial fractions derived from a CT loop gain Z-transform with those derived from a DT loop gain. This approach is intended to provide insight into the influence of each parameter (modulator order n and excess loop delay t_d) on the calculus to achieve the DT-CT equivalence and lead to a systematic method to compute the CT coefficients, Fig.(3).

[1]a proper fraction is a fraction which has the order of its denominator higher than the order of its numerator.

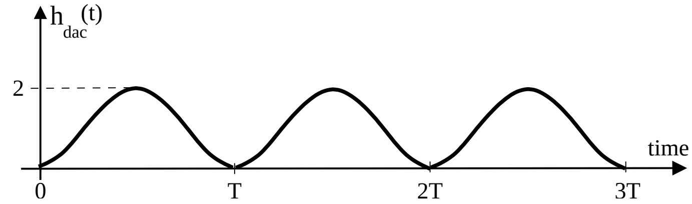

Fig. 2. Sine-shaped feedback signal ($h_{dac}(t) = 1 - \cos(w_{dac}t)$) with its period equal to the sampling period T.

In a first step we determine the expression of the DT loop gain in partial fractions.

Since we use passive resonators for the CT $\Sigma\Delta$ modulator, we consider all the resonators having the same center frequency ω_o. Hence, we place all the poles of the DT loop gain, which are the zeros of the NTF, at the same frequency without any optimization by spreading [11]. The DT loop gain have the following form [11]:

$$G_d(z) = \frac{\delta_{\frac{n}{2}-1}\ z^{\frac{n}{2}-1} + \delta_{\frac{n}{2}-2}\ z^{\frac{n}{2}-2} + \cdots + \delta_1\ z^1 + \delta_0}{(z^2+1)^{\frac{n}{2}}} \quad (7)$$

where δ_i are integer numbers. By expanding equation (7) in partial fractions we deduce the following expression :

$$G_d(z) = \sum_{k=1}^{\frac{n}{2}} \left(\frac{\epsilon_k}{(z-\jmath)^k} + \frac{\epsilon_k^*}{(z+\jmath)^k} \right) \quad (8)$$

ϵ_k are complex functions depending on δ_i and ϵ_k^* are their conjuguates as the $*$ indicates.

We have now to find in the partial fractions expressions of $H_u(z)$ and $H_c(z)$ the terms to do the equivalence with equation (8).

A. Useful Transfer Function

As shown in [7], a technique adapted to a systematic method to compute modified Z-transform is based on the residues:

$$\mathcal{Z}_{m_i}\{A(s)\} = \sum_{p_i \text{ poles of } A(s)} \text{Residues of } \frac{A(s)e^{m_i Ts}}{z-e^{Ts}} \quad (9)$$

By expanding in partial fractions the result of the modified Z-transform we find an expression where we can distinguish the terms due to the poles of the DAC and those due to the poles of the LC filter:

$$\mathcal{Z}_{m_i}\{A(s)\} = \frac{\beta_{dac}}{z-e^{\jmath\omega_{dac}T}} + \frac{\beta_{dac}^*}{z-e^{-\jmath\omega_{dac}T}} + \sum_{k=1}^{\frac{n}{2}} \left(\frac{\beta_k}{(z-e^{\jmath\omega_o T})^k} + \frac{\beta_k^*}{(z-e^{-\jmath\omega_o T})^k} \right) \quad (10)$$

where all the β are complex numbers and depend on m_i.

Considering that $\omega_{dac} = \frac{2\pi}{T}$ and $\omega_o = \frac{2\pi}{4T} = \frac{\pi}{2T}$, the equation

(10) becomes:

$$\mathcal{Z}_{m_i}\left\{A(s)\right\} = \frac{\beta_{dac}+\beta_{dac}^{*}}{z-1} + \sum_{k=1}^{\frac{n}{2}}\left(\frac{\beta_k}{(z-\jmath)^k} + \frac{\beta_k^{*}}{(z+\jmath)^k}\right) \quad (11)$$

From the above equation we can already notice that the terms which will be used to do the DT-CT equivalence are issued from the poles of the LC filter. This explains the link between the DT loop gain poles and the LC resonators center frequencies ω_o which has been discussed before.
Equation (11) in equation (5) leads to a form of $H_u(z)$ where we loose the useful terms to achieve the DT-CT equivalence:

$$H_U(z) = \sum_{i=0}^{M_u-1} u_i \left[\frac{\beta_{dac}+\beta_{dac}^{*}}{z^{n_i+1}} + \sum_{k=1}^{\frac{n}{2}}\left(\frac{\beta_k(z-1)}{z^{n_i+1}(z-\jmath)^k} + \frac{\beta_k^{*}(z-1)}{z^{n_i+1}(z+\jmath)^k}\right)\right] \quad (12)$$

Hence, we need to expand in partial fractions the equation (12):

$$H_U(z) = \underbrace{\sum_{i=1}^{D}\left(\frac{\gamma_{dac_i}+\gamma_{dac_i}^{*}+\gamma_{e_i}}{z^i}\right)}_{undesired\ terms} + \underbrace{\sum_{k=1}^{\frac{n}{2}}\left(\frac{\gamma_k}{(z-\jmath)^k} + \frac{\gamma_k^{*}}{(z+\jmath)^k}\right)}_{DT-CT\ equivalence} \quad (13)$$

where,

$$D = n_{M_u-1}+1 = \lfloor\frac{M_u-1}{2}+\frac{t_d}{T}\rfloor+1$$

and all the γ are complex numbers depending on m_i and coefficients u_i of the useful FIRDAC.
In equation (13) we have distinguished terms to identify with the DT loop gain from terms to cancel with $H_c(z)$. The identification with the DT loop gain defines a system of equations based on the numerators of the partial fractions. This leads to determine the values of useful FIRDAC coefficients u_i.

B. Compensation Transfer Function

Starting from equation (6) we use residues technique to compute the modified Z-transform of $B(s)$:

$$\begin{aligned}\mathcal{Z}_{m_i}\left\{B(s)\right\} &= \frac{1}{z-1} - \frac{e^{m_iT\jmath\omega_{dac}}+e^{-m_iT\jmath\omega_{dac}}}{2(z-1)} \\ &= \frac{1-cos(m_iT\omega_{dac})}{z-1}\end{aligned} \quad (14)$$

Thus, with $\omega_{dac}=\frac{2\pi}{T}$ the expression of $H_C(z)$ is:

$$H_C(z) = \sum_{i=0}^{i=M_c-1}\frac{c_i(1-cos(m_i2\pi))}{z^{\ell_i+1}} \quad (15)$$

By identifying the equation (15) with the undesired terms of $H_U(z)$, equation (13), we determine a new system of equations based on the numerators of the partial fractions. This system of equations allows to find the values of the coefficients which have not been determined by the DT-CT equivalence.

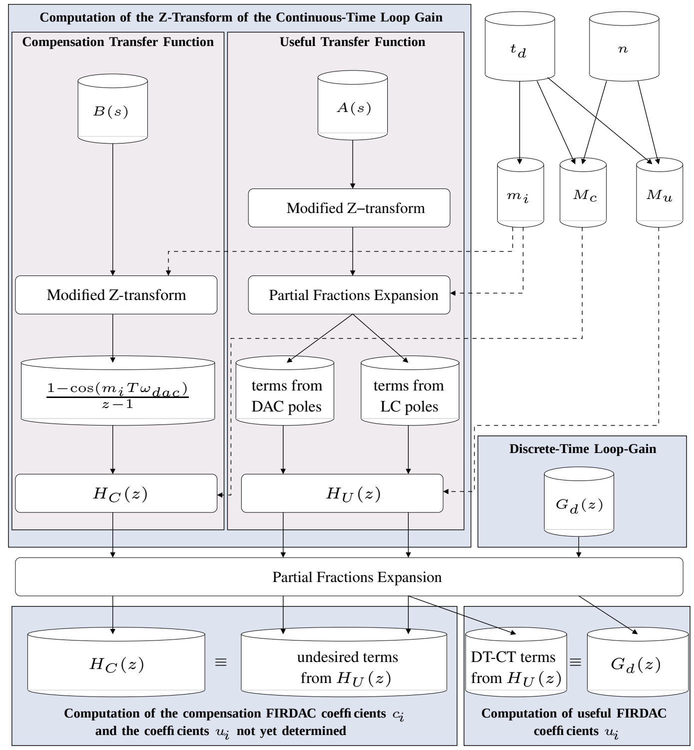

Fig. 3. Systematic approach to compute the CT coefficients for an LC $\Sigma\Delta$ modulator without intermediate nodes and using sine-shape feedbacks

C. Orders of the FIRDACs

The system derived from equation (13) defines the number of coefficients u_i needed to do the equivalence with the DT loop gain : n. When the excess loop delay is greater than 1 sampling period, the equation (15) shows that the compensation transfer function does not cancel all the undesired terms of the equation (13). Therefore, we have to use coefficients u_i of useful FIRDAC to cancel these terms and by doing this we have to increase the order of this FIRDAC. This leads to define the order of the useful FIRDAC as following:

$$M_u = n + \lfloor\frac{t_d}{T}\rfloor$$

Hence, we deduce the number of partial fractions that the compensation transfer function has to cancel:

$$n + D - M_u = D - \lfloor\frac{t_d}{T}\rfloor$$

By considering equation (15) we notice that some of the c_i coefficients can be nullify if m_i are integers. This depends on the value of t_d and in the worst case half of the coefficients c_i are nullified. Therefore, we double the number of c_i coefficients:

$$M_c = 2*(D - \lfloor\frac{t_d}{T}\rfloor)$$

In a favorable case, when the values of m_i are not integers the identification between the equation (15) and the undesired terms of $H_U(z)$ nullify the coefficients c_i in excess.
In another way, by increasing the order of the useful FIRDAC

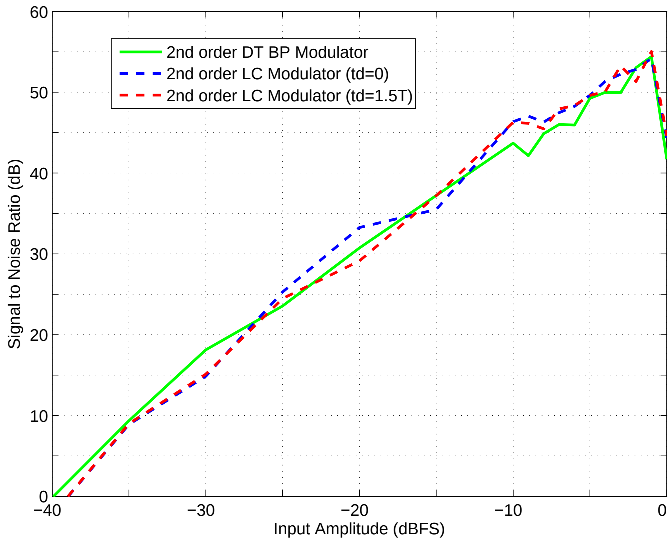

Fig. 4. Comparison of 2^{nd} order LC BP $\Sigma\Delta$ modulators with different excess loop delays (t_d) and a 2^{nd} order DT BP $\Sigma\Delta$ modulator (OSR=58, FFT points=16384)

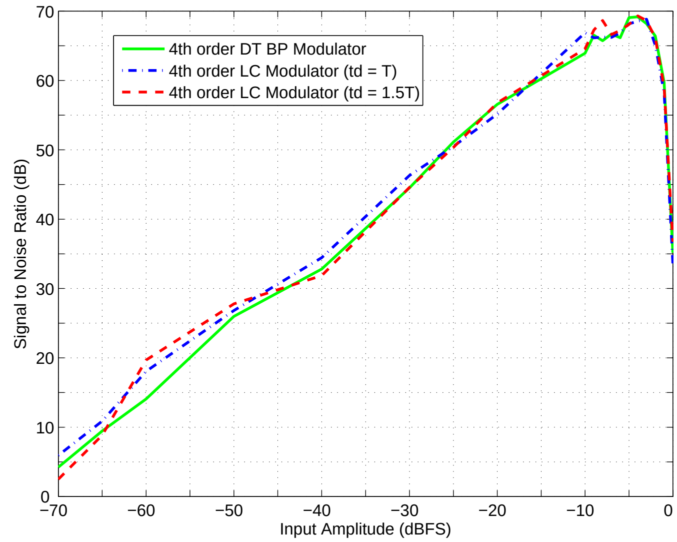

Fig. 5. Comparison of 4^{th} order LC BP $\Sigma\Delta$ modulators (Fig.1) with different excess loop delays (t_d) and a 4^{th} order DT BP $\Sigma\Delta$ modulator (OSR=58, FFT points=16384)

we can also offer additionnal degrees of freedom in order to relax some specifications [12].

IV. Design Examples

We have used the presented method to design second order and fourth order LC $\Sigma\Delta$ modulators with sine-shaped feedback. Different excess loop delays have been taken into account. The DT loop gains are derived from the transformation lowpass-to-bandpass, thus for a second order we have the following DT loop-gain :

$$G_d(z) = \frac{1}{z^2+1} \tag{16}$$

and for a fourth order :

$$G_d(z) = \frac{2z^2+1}{(z^2+1)^2} \tag{17}$$

The FIRDACs coefficients computed with the proposed method have been used to simulate under Matlab different modulators. The excess loop delay has been implemented in the modulator with a fixed delay placed before the FIRDACs, as shown in Fig.1. Hence, in Fig.4 we can compare the performance of a second order DT BP modulator and second order LC modulators with and without excess loop delay. The behavior is quite the same for the different modulators. This is confirmed by 4^{th} order modulators. We can see on Fig.5 that different 4^{th} order LC BP $\Sigma\Delta$ modulators achieve the same performance than a 4^{th} DT BP $\Sigma\Delta$ modulator.

V. Conclusion

In this paper, we presented a generalized technique for the determination of the CT coefficients for $\frac{f_s}{4}$ LC $\Sigma\Delta$ modulators using 2 feedback FIRDACs. This method is general and has been presented with architectures using sine-shaped feedback signals but it can be extended to rectangular feedback signals. The excess loop delay is taken into account without making more difficult the calculations since we have defined how to calculate the orders of the FIRDACs. Several examples of design have been simulated with different values of excess loop delay.

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